Diagnostische toets

bladzijde 165

1 a $x_P = p$ geeft $y_P = f(p) = \sqrt{8-2p}$

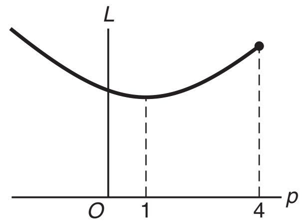
$L = OP = \sqrt{(x_P - 0)^2 + (y_P - 0)^2} = \sqrt{p^2 + 8 - 2p} = \sqrt{p^2 - 2p + 8}$

b $L = \sqrt{p^2 - 2p + 8} = (p^2 - 2p + 8)^{\frac{1}{2}}$ geeft $\dfrac{\mathrm{d}L}{\mathrm{d}p} = \frac{1}{2}(p^2 - 2p + 8)^{-\frac{1}{2}} \cdot (2p - 2) = \dfrac{p-1}{\sqrt{p^2 - 2p + 8}}$

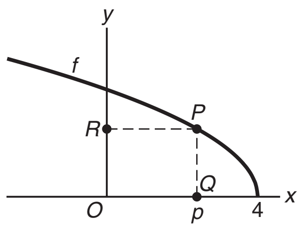
$\dfrac{\mathrm{d}L}{\mathrm{d}p} = 0$ geeft $p - 1 = 0$

$p = 1$

De minimale waarde van L is $\sqrt{1^2 - 2 \cdot 1 + 8} = \sqrt{7}$.



c



$A = O(OQPR) = OQ \cdot PQ = p\sqrt{8-2p} = p(8-2p)^{\frac{1}{2}}$

$\dfrac{\mathrm{d}A}{\mathrm{d}p} = 1 \cdot (8-2p)^{\frac{1}{2}} + p \cdot \frac{1}{2}(8-2p)^{-\frac{1}{2}} \cdot -2 = \sqrt{8-2p} - \dfrac{p}{\sqrt{8-2p}}$

$\dfrac{\mathrm{d}A}{\mathrm{d}p} = 0$ geeft $\sqrt{8-2p} - \dfrac{p}{\sqrt{8-2p}} = 0$

$\sqrt{8-2p} = \dfrac{p}{\sqrt{8-2p}}$

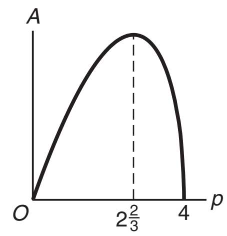
$p = 8 - 2p$

$3p = 8$

$p = 2\frac{2}{3}$

De maximale oppervlakte van rechthoek $OQPR$ is

$2\frac{2}{3}\sqrt{8 - 2 \cdot 2\frac{2}{3}} = 2\frac{2}{3}\sqrt{\frac{8}{3}} = 2\frac{2}{3}\sqrt{\frac{4}{9} \cdot 6} = 2\frac{2}{3} \cdot \frac{2}{3}\sqrt{6} = \frac{16}{9}\sqrt{6}$.

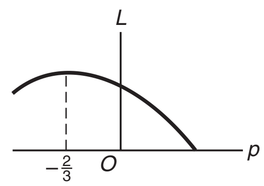


2 Uit de figuur blijkt dat voor de lengte L van CD geldt

$L = f(p) - g(p) = \sqrt{8 - 2p^2} - (\frac{1}{2}p + 2) = \sqrt{8 - 2p^2} - \frac{1}{2}p - 2 = (8 - 2p^2)^{\frac{1}{2}} - \frac{1}{2}p - 2$

$\frac{dL}{dp} = \frac{1}{2}(8 - 2p^2)^{-\frac{1}{2}} \cdot -4p - \frac{1}{2} = \frac{-2p}{\sqrt{8 - 2p^2}} - \frac{1}{2}$

$$\begin{aligned}\frac{dL}{dp} = 0 \text{ geeft } & \frac{-2p}{\sqrt{8 - 2p^2}} - \frac{1}{2} = 0 \\ & \frac{-2p}{\sqrt{8 - 2p^2}} = \frac{1}{2} \\ & -4p = \sqrt{8 - 2p^2} \\ & 16p^2 = 8 - 2p^2 \\ & 18p^2 = 8 \\ & p^2 = \frac{4}{9} \\ & p = -\frac{2}{3} \vee p = \frac{2}{3} \\ & \text{voldoet} \quad \text{voldoet niet}\end{aligned}$$



Dus de lengte van CD is maximaal voor $p = -\frac{2}{3}$.

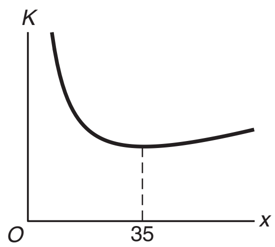
3 a K = kosten buitenste hekwerk + kosten binnenste hekwerken

$K = 160 \cdot (2x + 2y) + 85 \cdot 2y = 320x + 320y + 170y = 320x + 490y$

$$\left.\begin{array}{l} K = 320x + 490y \\ \left.\begin{array}{l} O = xy \\ O = 800 \end{array}\right\} xy = 800 \text{ dus } y = \frac{800}{x} \end{array}\right\} K = 320x + 490 \cdot \frac{800}{x} = 320x + \frac{392\,000}{x}$$

b $K = 320x + 392\,000x^{-1}$ geeft $\frac{dK}{dx} = 320 - 392\,000x^{-2} = 320 - \frac{392\,000}{x^2}$

$$\begin{aligned}\frac{dK}{dx} = 0 \text{ geeft } & 320 - \frac{392\,000}{x^2} = 0 \\ & 320 = \frac{392\,000}{x^2} \\ & 320x^2 = 392\,000 \\ & x^2 = 1225 \text{ dus } x = \sqrt{1225} = 35\end{aligned}$$

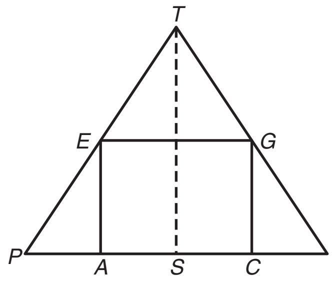


$x = 35$ geeft $y = \frac{800}{35} = 22\frac{6}{7}$

De kosten zijn minimaal bij de afmetingen 35 bij $22\frac{6}{7}$ m.

De minimale kosten zijn $320 \cdot 35 + \frac{392\,000}{35} = €\,22\,400$.

4 Stel $AB = a$, dan is $AC = a\sqrt{2}$ dus $AS = \frac{1}{2}a\sqrt{2}$.



$\triangle PAE \backsim \triangle PST$, dus $\dfrac{PA}{PS} = \dfrac{AE}{ST}$

$$\frac{4 - \frac{1}{2}a\sqrt{2}}{4} = \frac{AE}{6}$$

$$4AE = 24 - 3a\sqrt{2}$$

$$AE = 6 - \tfrac{3}{4}a\sqrt{2}$$

$I(\text{balk}) = AB \cdot BC \cdot AE = a \cdot a \cdot (6 - \frac{3}{4}a\sqrt{2}) = 6a^2 - \frac{3}{4}a^3 \cdot \sqrt{2}$

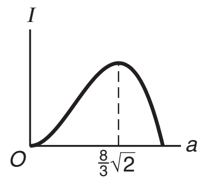
$\dfrac{dI}{da} = 12a - \frac{9}{4}a^2\sqrt{2}$

$\dfrac{dI}{da} = 0$ geeft $12a - \frac{9}{4}a^2 \cdot \sqrt{2} = 0$

$$a(12 - \tfrac{9}{4}a\sqrt{2}) = 0$$

$$a = 0 \vee \tfrac{9}{4}a\sqrt{2} = 12$$

$$a = 0 \vee a = \frac{12}{\frac{9}{4}\sqrt{2}} = \frac{12\sqrt{2}}{\frac{9}{2}} = \frac{24\sqrt{2}}{9} = \frac{8}{3}\sqrt{2}$$



Dus de maximale inhoud van de balk is

$6(\frac{8}{3}\sqrt{2})^2 - \frac{3}{4} \cdot (\frac{8}{3}\sqrt{2})^3 \cdot \sqrt{2} = \frac{256}{3} - \frac{512}{9} = \frac{256}{9}$.

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5 a Het snijpunt van AC en BD is S.
Omdat $AB = BC$ en $AD = CD$ is $AC \perp BD$.

In $\triangle ABS$ is $\cos(x) = \dfrac{BS}{5}$ dus $BS = 5\cos(x)$ en $\sin(x) = \dfrac{AS}{5}$ dus $AS = 5\sin(x)$.

In $\triangle ADS$ is $AS^2 + DS^2 = AD^2$

$$(5\sin(x))^2 + DS^2 = 6^2$$

$$25\sin^2(x) + DS^2 = 36$$

$$DS^2 = 36 - 25\sin^2(x) \text{ dus } DS = \sqrt{36 - 25\sin^2(x)}$$

$BD = BS + DS = 5\cos(x) + \sqrt{36 - 25\sin^2(x)}$

ALTERNATIEVE UITWERKING

De cosinusregel in $\triangle ABD$ geeft

$AD^2 = AB^2 + BD^2 - 2AB \cdot BD\cos(x)$

$36 = 25 + BD^2 - 10BD\cos(x)$

$BD^2 - 10BD\cos(x) - 11 = 0$

$D = (10\cos(x))^2 - 4 \cdot 1 \cdot -11 = 100\cos^2(x) + 44$

$BD = \dfrac{10\cos(x) - \sqrt{100\cos^2(x) + 44}}{2} \vee BD = \dfrac{10\cos(x) + \sqrt{100\cos^2(x) + 44}}{2}$

Omdat $BD > 0$ is $BD = \dfrac{10\cos(x) + \sqrt{100\cos^2(x) + 44}}{2}$

$$= 5\cos(x) + \tfrac{1}{2}\sqrt{4\left(25\cos^2(x) + 11\right)}$$

$$= 5\cos(x) + \sqrt{25\left(1 - \sin^2(x)\right) + 11}$$

$$= 5\cos(x) + \sqrt{25 - 25\sin^2(x) + 11}$$

$$= 5\cos(x) + \sqrt{36 - 25\sin^2(x)}$$

b $AS = 5\sin(x)$ dus $AC = 10\sin(x)$

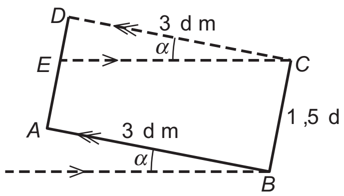
$BD = AC$ geeft $5\cos(x) + \sqrt{36 - 25\sin^2(x)} = 10\sin(x)$

Voer in $y_1 = 5\cos(x) + \sqrt{36 - 25\sin^2(x)}$ en $y_2 = 10\sin(x)$

De optie intersect geeft $x \approx 0{,}89$.

Dus voor $x \approx 0{,}89$ is $BD = AC$.

6 a



Stel de lengte van de goot is l dm.

$O = O(ABCE)$

$$\left.\begin{aligned} &I(\text{rechte goot}) = 3 \cdot 1{,}5 \cdot l = 4{,}5l \\ &I(\text{rechte goot}) = 180 \end{aligned}\right\} \begin{aligned} &4{,}5l = 180 \\ &l = 40 \end{aligned}$$

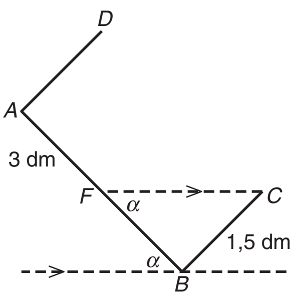
$$\left.\begin{aligned} &I(\text{gekantelde goot}) = O \cdot 40 \\ &I(\text{gekantelde goot}) = 144 \end{aligned}\right\} \begin{aligned} &40O = 144 \\ &O = 3{,}6 \end{aligned}$$

In $\triangle CDE$ is $\tan\alpha = \dfrac{DE}{CD}$

$$\tan\alpha = \frac{DE}{3} \text{ dus } DE = 3\tan\alpha$$

$$\left.\begin{aligned} &O = O(ABCD) - O(CDE) = 3 \cdot 1{,}5 - \tfrac{1}{2} \cdot 3 \cdot 3\tan\alpha = 4{,}5 - 4{,}5\tan\alpha \\ &O = 3{,}6 \end{aligned}\right\} \begin{aligned} &4{,}5 - 4{,}5\tan\alpha = 3{,}6 \\ &4{,}5\tan\alpha = 0{,}9 \\ &\tan\alpha = 0{,}2 \\ &\alpha \approx 11^\circ \end{aligned}$$

b



$$\left.\begin{aligned} &I(\text{gekantelde goot}) = O(BCF) \cdot 40 \\ &I(\text{gekantelde goot}) = 45 \end{aligned}\right\} \begin{aligned} &40 \cdot O(BCF) = 45 \\ &O(BCF) = 1\tfrac{1}{8} \end{aligned}$$

In $\triangle BCF$ is $\tan\alpha = \dfrac{BC}{BF}$

$$\tan\alpha = \frac{1{,}5}{BF}$$

$$BF = \frac{1{,}5}{\tan\alpha}$$

$$O(BCF) = \tfrac{1}{2} \cdot BC \cdot BF = \tfrac{1}{2} \cdot 1{,}5 \cdot \frac{1{,}5}{\tan\alpha} = \frac{1{,}125}{\tan\alpha}$$

$O(BCF) = 1\tfrac{1}{8}$ geeft $\dfrac{1{,}125}{\tan\alpha} = 1\tfrac{1}{8}$

$$\tan\alpha = 1$$

$$\alpha = 45^\circ$$

Dus vanaf een kantelhoek van 45° past er minder dan 45 liter in de goot.

7 a $v(3) = v(0) + \int_0^3 a(p)\mathrm{d}p$ geeft $8 = v(0) + \int_0^3 2\ \mathrm{d}p$

$$8 = v(0) + [2p]_0^3$$

$$8 = v(0) + 6 - 0$$

$$v(0) = 2$$

Dus de snelheid op $t = 0$ is 2 cm/s.

ALTERNATIEVE UITWERKING

$$\left.\begin{array}{l} a(t) = 2 \text{ geeft } v(t) = 2t + b \\ v(3) = 8 \end{array}\right\} \begin{array}{l} 2 \cdot 3 + b = 8 \\ 6 + b = 8 \\ b = 2 \end{array}$$

Dus $v(t) = 2t + 2$.

De snelheid op $t = 0$ is 2 cm/s.

b $v(t) = v(0) + \int_0^t a(p)\mathrm{d}p = 2 + \int_0^t 2\ \mathrm{d}p = 2 + [2p]_0^t = 2 + 2t - 0 = 2 + 2t$

$$\int_0^{10} v(p)\mathrm{d}p = \int_0^{10} (2 + 2p)\mathrm{d}p = \left[2p + p^2\right]_0^{10} = (2 \cdot 10 + 10^2) - 0 = 120$$

De afgelegde weg in de eerste 10 seconden is 120 cm.

ALTERNATIEVE UITWERKING

$v(t) = 2t + 2$ geeft $s(t) = t^2 + 2t + c$

$s(0) = c$ en $s(10) = 10^2 + 2 \cdot 10 + c = 120 + c$, dus de afgelegde weg in de eerste 10 seconden is 120 cm.

8 $$v_{\text{gem}} = \frac{s(b) - s(a)}{b - a} = \frac{\int_a^b s'(t)\mathrm{d}t}{b - a} = \frac{\int_a^b v(t)\mathrm{d}t}{b - a}$$

9 $O(V) = \int_0^1 3^x \, dx \approx 1{,}820$

$\int_0^1 x \cdot 3^x \, dx \approx 1{,}074$

$x_Z \approx \frac{1{,}074}{1{,}820} \approx 0{,}59$

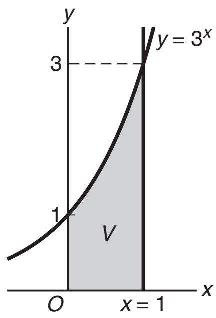
$y = 3^x$ geeft $x = {}^3\log(y)$

$O(V) = \int_0^1 1 \, dy + \int_1^3 \left(1 - {}^3\log(y)\right) dy \approx 1{,}820$

$\int_0^1 y \cdot 1 \, dy + \int_1^3 y\left(1 - {}^3\log(y)\right) dy \approx 1{,}820$

$y_Z \approx \frac{1{,}820}{1{,}820} = 1$

Dus $Z(0{,}59; 1)$.



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10 a $f(x) = \frac{3}{5 - x} = 3 \cdot \frac{1}{5 - x}$ geeft $F(x) = 3 \cdot \frac{1}{-1} \ln|5 - x| = -3 \ln|5 - x|$

b $g(x) = e^{2x+1}$ geeft $G(x) = \frac{1}{2} e^{2x+1}$

c $h(x) = 2 \ln(8 - x)$ geeft

$H(x) = 2\left(\frac{1}{-1}\left((8 - x)\ln(8 - x) - (8 - x)\right)\right) = (2x - 16)\ln(8 - x) + 16 - 2x$

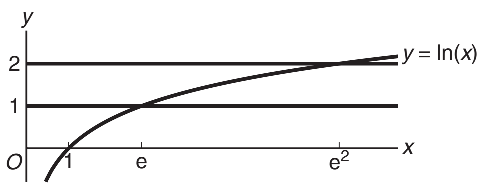
d $j(x) = 5 \sin(2\pi x)$ geeft $J(x) = 5 \cdot \frac{1}{2\pi} \cdot -\cos(2\pi x) = -\frac{5}{2\pi}\cos(2\pi x)$

e $k(x) = (3x - 1) \cdot \sqrt[3]{3x - 1} = (3x - 1)^{1\frac{1}{3}}$ geeft

$K(x) = \frac{1}{3} \cdot \frac{1}{2\frac{1}{3}}(3x - 1)^{2\frac{1}{3}} = \frac{1}{7}(3x - 1)^2 \cdot \sqrt[3]{3x - 1}$

f $l(x) = 0{,}5^{1 - 0{,}5x}$ geeft $L(x) = \frac{1}{-0{,}5} \cdot \frac{0{,}5^{1-0{,}5x}}{\ln(0{,}5)} = \frac{-2 \cdot 0{,}5^{1-0{,}5x}}{\ln(0{,}5)}$

11

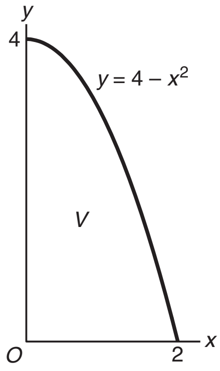


$\ln(x) = 0$	$\ln(x) = 1$	$\ln(x) = 2$
$x = 1$	$x = e$	$x = e^2$

Dus $\text{int}(\ln(x)) = \begin{cases} 0 \text{ voor } 1 \le x < e \\ 1 \text{ voor } e \le x < e^2 \\ 2 \text{ voor } x = e^2 \end{cases}$

$\int_1^{e^2} \text{int}(\ln(x))dx = \int_1^e 0 \, dx + \int_e^{e^2} 1 \, dx = 0 + [x]_e^{e^2} = e^2 - e$

12



a $I(L) = \int_0^2 \pi(4 - x^2)^2 \,dx = \int_0^2 \pi(16 - 8x^2 + x^4)dx = \left[\pi(16x - \frac{8}{3}x^3 + \frac{1}{5}x^5)\right]_0^2$

$= \pi(32 - \frac{64}{3} + \frac{32}{5}) - 0 = \frac{256}{15}\pi$

b $y = 4 - x^2$ geeft $x^2 = 4 - y$

$I(M) = \int_0^4 \pi(4 - y)dy = \left[\pi\left(4y - \frac{1}{2}y^2\right)\right]_0^4 = \pi(16 - 8) - 0 = 8\pi$

c $\int_0^2 \pi x y^2 \,dx = \int_0^2 \pi x(16 - 8x^2 + x^4)dx = \int_0^2 \pi(16x - 8x^3 + x^5)dx$

$= \left[\pi\left(8x^2 - 2x^4 + \frac{1}{6}x^6\right)\right]_0^2 = \pi(32 - 32 + \frac{64}{6}) - 0 = \frac{32}{3}\pi$

De x-coördinaat van het zwaartepunt van L is $\dfrac{\frac{32}{3}\pi}{\frac{256}{15}\pi} = \frac{5}{8}$.

d $\int_0^4 \pi y x^2 \,dy = \int_0^4 \pi y(4 - y)dy = \int_0^4 \pi(4y - y^2)dy = \left[\pi\left(2y^2 - \frac{1}{3}y^3\right)\right]_0^4$

$= \pi(32 - \frac{64}{3}) - 0 = \frac{32}{3}\pi$

De y-coördinaat van het zwaartepunt van M is $\dfrac{\frac{32}{3}\pi}{8\pi} = 1\frac{1}{3}$.

13 Stel er zitten x blauwe knikkers in de vaas.

$P(\text{2 rode, 2 witte en 2 blauwe}) = \dfrac{\binom{3}{2} \cdot \binom{5}{2} \cdot \binom{x}{2}}{\binom{8+x}{6}}$

Voer in $y_1 = \dfrac{3 \cdot \binom{5}{2} \cdot \binom{x}{2}}{\binom{8+x}{6}}$.

a Voor $x = 11$ is $y \approx 0{,}06$, dus er zitten 11 blauwe knikkers in de vaas.

b Voor $x = 2$ is $y \approx 0{,}14$; voor $x = 3$ is $y \approx 0{,}19$; voor $x = 4$ is $y \approx 0{,}19$ en voor $x = 5$ is $y \approx 0{,}17$.

Dus de kans op twee rode, twee witte en twee blauwe knikkers is het grootst bij 3 en bij 4 blauwe knikkers.

14 X = het aantal mensen dat komt opdagen

$P(\text{er is niet genoeg plaats}) = P(X > 482) = 1 - P(X \leq 482)$

Dus $1 - P(X \leq 482) \leq 0{,}10$ ofwel $P(X \leq 482) \geq 0{,}9$.

binomcdf(506, 0.94, 482) $\approx 0{,}904$ en binomcdf(507, 0.94, 482) $\approx 0{,}868$

Er zijn dus maximaal 506 reserveringen toegestaan.

15 a $P(18 \text{ keer gooien}) = P(\text{bij 17 keer gooien drie keer een 6}) \cdot P(\text{een 6})$

$$= \binom{17}{3} \cdot \left(\tfrac{1}{6}\right)^3 \cdot \left(\tfrac{5}{6}\right)^{14} \cdot \tfrac{1}{6} \approx 0{,}041$$

b $P(n) = P(\text{bij } n-1 \text{ keer gooien drie keer een 6}) \cdot P(\text{een 6})$

$$= \binom{n-1}{3} \cdot \left(\tfrac{1}{6}\right)^3 \cdot \left(\tfrac{5}{6}\right)^{n-1-3} \cdot \tfrac{1}{6} = \binom{n-1}{3} \cdot \left(\tfrac{1}{6}\right)^4 \cdot \left(\tfrac{5}{6}\right)^{n-4}$$

Voer in $y_1 = \binom{x-1}{3} \cdot \left(\tfrac{1}{6}\right)^4 \cdot \left(\tfrac{5}{6}\right)^{x-4}$ en lees af in de tabel:

$P(17) \approx 0{,}0404$, $P(18) \approx 0{,}0409$, $P(19) \approx 0{,}0409$ en $P(20) \approx 0{,}0404$

Dus bij 18 en 19 keer gooien is de kans om bij de laatste worp de vierde 6 te gooien maximaal.

c $P(7) \approx 0{,}009$ en $P(8) \approx 0{,}013$; $P(39) \approx 0{,}01102$ en $P(40) \approx 0{,}00995$

Dus voor $n = 1, 2, 3, \ldots, 7$ en $n = 40, 41, 42, \ldots$ is de kans om bij de laatste worp de vierde 6 te gooien kleiner dan 0,01.