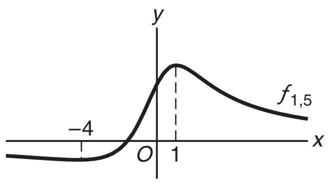
Dit geeft $f_{1,5}(x) = \dfrac{x + 1{,}5}{x^2 + 4}$ en $f'_{1,5}(x) = \dfrac{-x^2 - 3x + 4}{(x^2 + 4)^2}$



$$\begin{aligned} f'_{1,5}(x) = 0 \text{ geeft } & -x^2 - 3x + 4 = 0 \\ & x^2 + 3x - 4 = 0 \\ & (x + 4)(x - 1) = 0 \\ & x = -4 \vee x = 1 \end{aligned}$$

De andere extreme waarde: min. is $f_{1,5}(-4) = -0{,}125$.

c $f'_p(x) = 0$ geeft $-x^2 - 2px + 4 = 0$

$$-2px = x^2 - 4$$

$$\left.\begin{aligned} p &= \frac{x^2 - 4}{-2x} \\ \text{Verder is } y &= \frac{x + p}{x^2 + 4} \end{aligned}\right\} y = \frac{x + \dfrac{x^2 - 4}{-2x}}{x^2 + 4} = \frac{-2x^2 + x^2 - 4}{-2x(x^2 + 4)}$$

$$= \frac{-x^2 - 4}{-2x(x^2 + 4)} = \frac{x^2 + 4}{2x(x^2 + 4)} = \frac{1}{2x}$$

Dus alle toppen liggen op de kromme $y = \dfrac{1}{2x}$.

Diagnostische toets

bladzijde 34

1 $f(x) = 5x^2 + 4$

a
$$\begin{aligned} f'(3) &= \lim_{h \to 0} \frac{f(3 + h) - f(3)}{h} \\ &= \lim_{h \to 0} \frac{5(3 + h)^2 + 4 - (5 \cdot 3^2 + 4)}{h} \\ &= \lim_{h \to 0} \frac{5(9 + 6h + h^2) + 4 - 49}{h} \\ &= \lim_{h \to 0} \frac{45 + 30h + 5h^2 - 45}{h} \\ &= \lim_{h \to 0} \frac{30h + 5h^2}{h} \\ &= \lim_{h \to 0} (30 + 5h) = 30 \end{aligned}$$

b
$$\begin{aligned} f'(x) &= \lim_{h \to 0} \frac{f(x + h) - f(x)}{h} \\ &= \lim_{h \to 0} \frac{5(x + h)^2 + 4 - (5x^2 + 4)}{h} \\ &= \lim_{h \to 0} \frac{5(x^2 + 2xh + h^2) + 4 - 5x^2 - 4}{h} \\ &= \lim_{h \to 0} \frac{5x^2 + 10xh + 5h^2 - 5x^2}{h} \\ &= \lim_{h \to 0} \frac{10xh + 5h^2}{h} \\ &= \lim_{h \to 0} (10x + 5h) = 10x \end{aligned}$$

2 a
$$\begin{aligned} f(x) &= (x^2 + 3x)(3 - 7x) \text{ geeft} \\ f'(x) &= [x^2 + 3x]' \cdot (3 - 7x) + (x^2 + 3x) \cdot [3 - 7x]' \\ &= (2x + 3)(3 - 7x) + (x^2 + 3x) \cdot -7 \\ &= (2x + 3)(3 - 7x) - 7(x^2 + 3x) \end{aligned}$$

b
$$\begin{aligned} g(x) &= (3x^2 + 4)^2 = (3x^2 + 4)(3x^2 + 4) \text{ geeft} \\ g'(x) &= [3x^2 + 4]' \cdot (3x^2 + 4) + (3x^2 + 4) \cdot [3x^2 + 4]' \\ &= 6x(3x^2 + 4) + (3x^2 + 4) \cdot 6x \\ &= 12x(3x^2 + 4) \end{aligned}$$

3 a $f(x) = \dfrac{3x-7}{x^2+2}$ geeft

$$f'(x) = \frac{(x^2+2)\cdot[3x-7]' - (3x-7)\cdot[x^2+2]'}{(x^2+2)^2} = \frac{(x^2+2)\cdot 3 - (3x-7)\cdot 2x}{(x^2+2)^2}$$

$$= \frac{3x^2+6-6x^2+14x}{(x^2+2)^2} = \frac{-3x^2+14x+6}{(x^2+2)^2}$$

b $g(x) = 3x - \dfrac{2}{x+4}$ geeft

$$g'(x) = 3 - \frac{(x+4)\cdot[2]' - 2\cdot[x+4]'}{(x+4)^2} = 3 - \frac{(x+4)\cdot 0 - 2\cdot 1}{(x+4)^2} = 3 + \frac{2}{(x+4)^2}$$

4 $f(x) = \dfrac{x^2-9}{3x+2}$ geeft

$$f'(x) = \frac{(3x+2)\cdot[x^2-9]' - (x^2-9)\cdot[3x+2]'}{(3x+2)^2} = \frac{(3x+2)\cdot 2x - (x^2-9)\cdot 3}{(3x+2)^2}$$

$$= \frac{6x^2+4x-3x^2+27}{(3x+2)^2} = \frac{3x^2+4x+27}{(3x+2)^2}$$

$f(x) = 0$ geeft $\dfrac{x^2-9}{3x+2} = 0$

$x^2 - 9 = 0$

$x^2 = 9$

$x = -3 \vee x = 3$

Dus $A(-3, 0)$ en $B(3, 0)$.

Stel $k: y = ax + b$ met $a = f'(-3) = \dfrac{27-12+27}{(-7)^2} = \frac{6}{7}$.

$\left.\begin{array}{l} k: y = \frac{6}{7}x + b \\ A(-3, 0) \end{array}\right\}$ $0 = \frac{6}{7}\cdot -3 + b$

$\frac{18}{7} = b$

Dus $k: y = \frac{6}{7}x + \frac{18}{7}$.

Stel $l: y = ax + b$ met $a = f'(3) = \dfrac{27+12+27}{11^2} = \frac{6}{11}$.

$\left.\begin{array}{l} l: y = \frac{6}{11}x + b \\ B(3, 0) \end{array}\right\}$ $0 = \frac{6}{11}\cdot 3 + b$

$-\frac{18}{11} = b$

Dus $l: y = \frac{6}{11}x - \frac{18}{11}$.

5 a $f(x) = \dfrac{2}{x^5} = 2x^{-5}$ geeft $f'(x) = -10x^{-6} = -\dfrac{10}{x^6}$

b $g(x) = \dfrac{x^5+2}{x^3} = \dfrac{x^5}{x^3} + \dfrac{2}{x^3} = x^2 + 2x^{-3}$ geeft $g'(x) = 2x - 6x^{-4} = 2x - \dfrac{6}{x^4}$

c $h(x) = \dfrac{3}{x} - \dfrac{x}{3} = 3x^{-1} - \frac{1}{3}x$ geeft $h'(x) = -3x^{-2} - \frac{1}{3} = -\dfrac{3}{x^2} - \frac{1}{3}$

6 a $f(x) = x^3 + \sqrt[3]{x^2} = x^3 + x^{\frac{2}{3}}$ geeft $f'(x) = 3x^2 + \frac{2}{3}x^{-\frac{1}{3}} = 3x^2 + \frac{2}{3}\cdot\dfrac{1}{x^{\frac{1}{3}}} = 3x^2 + \dfrac{2}{3\cdot\sqrt[3]{x}}$

b $g(x) = x^3 \cdot \sqrt[3]{x^2} = x^3 \cdot x^{\frac{2}{3}} = x^{3\frac{2}{3}}$ geeft $g'(x) = 3\frac{2}{3}x^{2\frac{2}{3}} = 3\frac{2}{3}x^2 \cdot \sqrt[3]{x^2}$

c $h(x) = \dfrac{x\sqrt{x}}{x^3+1} = \dfrac{x^{1\frac{1}{2}}}{x^3+1}$ geeft

$$h'(x) = \frac{(x^3+1)\cdot[x^{1\frac{1}{2}}]' - x^{1\frac{1}{2}}\cdot[x^3+1]'}{(x^3+1)^2} = \frac{(x^3+1)\cdot 1\frac{1}{2}x^{\frac{1}{2}} - x^{1\frac{1}{2}}\cdot 3x^2}{(x^3+1)^2}$$

$$= \frac{1\frac{1}{2}x^{3\frac{1}{2}} + 1\frac{1}{2}x^{\frac{1}{2}} - 3x^{3\frac{1}{2}}}{(x^3+1)^2} = \frac{-1\frac{1}{2}x^{3\frac{1}{2}} + 1\frac{1}{2}x^{\frac{1}{2}}}{(x^3+1)^2} = \frac{-1\frac{1}{2}x^3\cdot\sqrt{x} + 1\frac{1}{2}\sqrt{x}}{(x^3+1)^2}$$

7 $f(x)=\frac{x^2-3}{x^2\cdot\sqrt{x}}=\frac{x^2-3}{x^{2\frac{1}{2}}}=\frac{x^2}{x^{2\frac{1}{2}}}-\frac{3}{x^{2\frac{1}{2}}}=x^{-\frac{1}{2}}-3x^{-2\frac{1}{2}}$ geeft

$f'(x)=-\frac{1}{2}x^{1\frac{1}{2}}+7\frac{1}{2}x^{-3\frac{1}{2}}=-\frac{1}{2x^{1\frac{1}{2}}}+\frac{15}{2x^{3\frac{1}{2}}}=-\frac{1}{2x\sqrt{x}}+\frac{15}{2x^3\cdot\sqrt{x}}$

Stel $k\colon y=ax+b$ met $a=f'(1)=-\frac{1}{2}+\frac{15}{2}=7.$

$$\left.\begin{array}{l} k\colon y=7x+b \\ y_A=f(1)=\frac{1-3}{1}=-2 \text{ dus } A(1,\ -2) \end{array}\right\} \begin{array}{l} -2=7\cdot1+b \\ -9=b \end{array}$$

Dus $k\colon y=7x-9$.

$y=0$ geeft $0=7x-9$

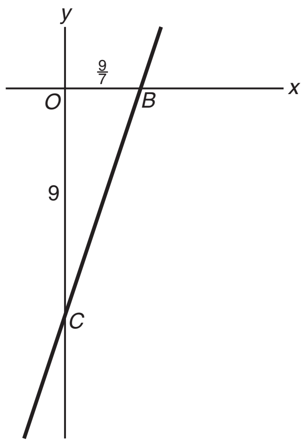
$9=7x$

$x=\frac{9}{7}$

Dus $B(\frac{9}{7},0)$.

$x=0$ geeft $y=0-9=-9$

Dus $C(0,\ -9)$.



$O(\triangle OBC)=\frac{1}{2}\cdot\frac{9}{7}\cdot9=\frac{81}{14}$

8 a $f(x)=\frac{-x}{x^2+1}$ geeft

$f'(x)=\frac{(x^2+1)\cdot-1--x\cdot2x}{(x^2+1)^2}=\frac{-x^2-1+2x^2}{(x^2+1)^2}=\frac{x^2-1}{(x^2+1)^2}$

$f'(x)=0$ geeft $\frac{x^2-1}{(x^2+1)^2}=0$

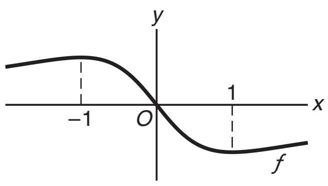
$x^2-1=0$

$x^2=1$

$x=-1 \vee x=1$

max. is $f(-1)=\frac{1}{1+1}=\frac{1}{2}$

min. is $f(1)=\frac{-1}{1+1}=-\frac{1}{2}$



b $f'(x) = \frac{3}{25}$ geeft

$\frac{x^2 - 1}{(x^2 + 1)^2} = \frac{3}{25}$

$3(x^2 + 1)^2 = 25(x^2 - 1)$

$3(x^4 + 2x^2 + 1) = 25x^2 - 25$

$3x^4 + 6x^2 + 3 - 25x^2 + 25 = 0$

$3x^4 - 19x^2 + 28 = 0$

Stel $x^2 = p$.

$3p^2 - 19p + 28 = 0$

$D = (-19)^2 - 4 \cdot 3 \cdot 28 = 25$

$p = \frac{19 - 5}{6} \vee p = \frac{19 + 5}{6}$

$p = \frac{7}{3} \vee p = 4$

$x^2 = \frac{7}{3} \vee x^2 = 4$

$x = -\sqrt{\frac{7}{3}} \vee x = \sqrt{\frac{7}{3}} \vee x = -2 \vee x = 2$

bladzijde 35

9 $h = -5t^2 + 30t$ geeft $\frac{dh}{dt} = -10t + 30$

$\frac{dh}{dt} = -5$ geeft $-10t + 30 = -5$

$-10t = -35$

$t = 3{,}5$

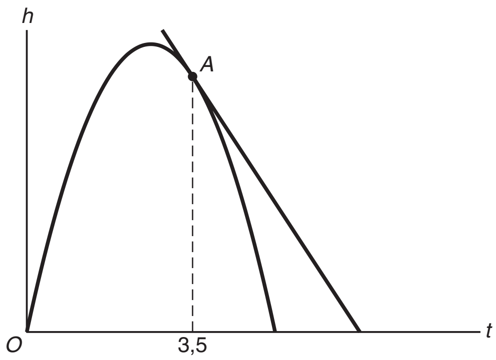
$t = 3{,}5$ geeft $h = -5 \cdot 3{,}5^2 + 30 \cdot 3{,}5 = 43{,}75$ m

Vanaf $t = 3{,}5$ is de snelheid 5 m/s dus er verloopt vanaf $t = 3{,}5$ een tijd van

$\frac{43{,}75}{5} = 8{,}75$ seconden voor de pijl op de grond is.

Dus $3{,}5 + 8{,}75 = 12{,}25$ seconden na het afschieten is de pijl weer op de grond.

ALTERNATIEVE UITWERKING



$h = -5t^2 + 30t$ geeft $\frac{dh}{dt} = -10t + 30$

$\frac{dh}{dt} = -5$ geeft $-10t + 30 = -5$

$-10t = -35$

$t = 3{,}5$

$t = 3{,}5$ geeft $h = -5 \cdot 3{,}5^2 + 30 \cdot 3{,}5 = 43{,}75$

Dus $A(3{,}5; 43{,}75)$.

Stel raaklijn in A is $h = at + b$ met $a = -5$

$\left.\begin{array}{l} h = -5t + b \\ A(3{,}5; 43{,}75) \end{array}\right\} 43{,}75 = -5 \cdot 3{,}5 + b$

$43{,}75 + 17{,}5 = b$

$61{,}25 = b$

Dus raaklijn in A is $h = -5t + 61{,}25$.

$h = 0$ geeft $0 = -5t + 61{,}25$

$5t = 61{,}25$

$t = 12{,}25$

Dus na 12,25 seconden weer op de grond.

10 a $f(x) = x^3 - 4x^2 + 4x + 3$ geeft

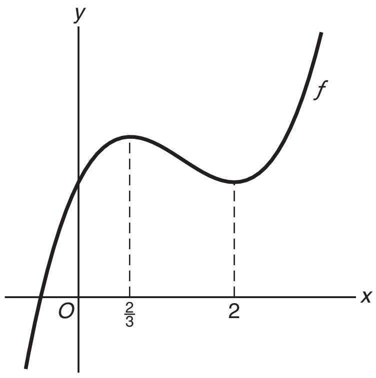
$f'(x) = 3x^2 - 8x + 4$

$f'(x) = 0$ geeft $3x^2 - 8x + 4 = 0$

$D = (-8)^2 - 4 \cdot 3 \cdot 4 = 16$

$x = \dfrac{8+4}{6} \vee x = \dfrac{8-4}{6}$

$x = 2 \vee x = \frac{2}{3}$



max. is $f(\frac{2}{3}) = \frac{113}{27}$

min. is $f(2) = 3$

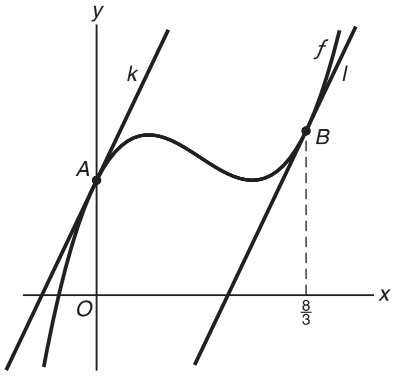
b $f(x) = p$ heeft precies één oplossing voor $p < 3 \vee p > \frac{113}{27}$.

c $f'(x) = 4$ geeft $3x^2 - 8x + 4 = 4$

$3x^2 - 8x = 0$

$x(3x - 8) = 0$

$x = 0 \vee x = \frac{8}{3}$



$f(0) = 3$ dus $A(0, 3)$.

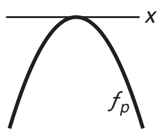
Dus $k{:}\ y = 4x + 3$.

$$\left.\begin{array}{l} l{:}\ y = 4x + b \\ f(\frac{8}{3}) = \frac{113}{27} \text{ dus } B(\frac{8}{3},\ \frac{113}{27}) \end{array}\right\} \begin{array}{l} \frac{113}{27} = 4 \cdot \frac{8}{3} + b \\ -\frac{175}{27} = b \end{array}$$

Dus $l{:}\ y = 4x - \frac{175}{27}$.

Dus $f(x) = 4x + p$ heeft precies één oplossing voor $p < -\frac{175}{27} \vee p > 3$.

11 a



$f_p(x) = -x^2 + px - 3$ heeft top op de x-as als $D = 0$.

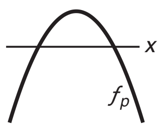
Dit geeft $p^2 - 4 \cdot -1 \cdot -3 = 0$

$p^2 - 12 = 0$

$p^2 = 12$

$p = -\sqrt{12} \vee p = \sqrt{12}$

b



f_p heeft een positief maximum voor $D > 0$.

Dit geeft $p^2 - 12 > 0$

$p^2 > 12$

$p < -\sqrt{12} \vee p > \sqrt{12}$

12 $f_p(x) = -\frac{1}{3}x^3 + 2x^2 - px + 5$ geeft $f_p'(x) = -x^2 + 4x - p$

f_p heeft twee extremen als $f_p'(x) = 0$ twee oplossingen heeft, dus als $D > 0$. Dit geeft

$16 - 4 \cdot -1 \cdot -p > 0$

$16 - 4p > 0$

$-4p > -16$

$p < 4$

13 $f_p(x) = \dfrac{2\sqrt{x} + p}{x + 1}$ geeft

$$f_p'(x) = \frac{(x+1) \cdot [2\sqrt{x} + p]' - (2\sqrt{x} + p) \cdot [x+1]'}{(x+1)^2} = \frac{(x+1) \cdot 2 \cdot \frac{1}{2\sqrt{x}} - (2\sqrt{x} + p) \cdot 1}{(x+1)^2}$$

$$= \frac{(x+1) \cdot \frac{1}{\sqrt{x}} - 2\sqrt{x} - p}{(x+1)^2} = \frac{x + 1 - 2x - p\sqrt{x}}{(x+1)^2 \cdot \sqrt{x}} = \frac{-x - p\sqrt{x} + 1}{(x+1)^2 \cdot \sqrt{x}}$$

$f_p'(4) = -0{,}1$ geeft $\dfrac{-4 - 2p + 1}{(4+1)^2 \cdot 2} = -0{,}1$

$$\frac{-3 - 2p}{50} = -0{,}1$$

$$-3 - 2p = -5$$

$$-2p = -2$$

$$p = 1$$

$f_1(x) = \dfrac{2\sqrt{x} + 1}{x + 1}$

$$\left.\begin{array}{l} f_1(4) = \dfrac{4+1}{4+1} = 1 \text{ dus } A(4, 1) \\ k: y = -0{,}1x + q \end{array}\right\} \begin{array}{l} 1 = -0{,}1 \cdot 4 + q \\ 1{,}4 = q \end{array}$$

Dus $p = 1$ en $q = 1{,}4$.

14 $f_p(x) = -\frac{1}{3}x^3 + px^2 + 3x - 4$ geeft $f_p'(x) = -x^2 + 2px + 3$

$f_p'(x) = 0$ geeft $-x^2 + 2px + 3 = 0$

$$2px = x^2 - 3$$

$$p = \frac{x^2 - 3}{2x}$$

$p = \dfrac{x^2 - 3}{2x}$ geeft $y = -\frac{1}{3}x^3 + \dfrac{x^2 - 3}{2x} \cdot x^2 + 3x - 4$

$$= -\tfrac{1}{3}x^3 + \frac{x^2 - 3}{2} \cdot x + 3x - 4$$

$$= -\tfrac{1}{3}x^3 + \tfrac{1}{2}x^3 - 1\tfrac{1}{2}x + 3x - 4$$

$$= \tfrac{1}{6}x^3 + 1\tfrac{1}{2}x - 4$$

Dus de gevraagde formule is $y = \frac{1}{6}x^3 + 1\frac{1}{2}x - 4$.