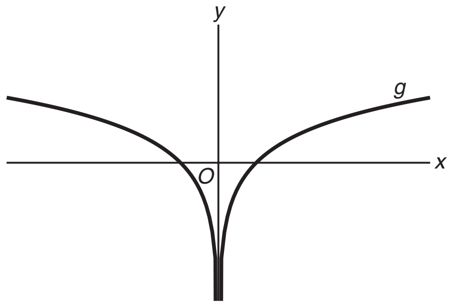
f $g(x) = \ln|x| = \begin{cases} \ln(x) & \text{voor } x > 0 \\ \ln(-x) & \text{voor } x < 0 \end{cases}$

Dit geeft $g'(x) = \begin{cases} \frac{1}{x} & \text{voor } x > 0 \\ \frac{1}{x} & \text{voor } x < 0 \end{cases}$

ofwel $g'(x) = \frac{1}{x}$.



Diagnostische toets

bladzijde 138

1 a $\left(\frac{2}{3}\right)^{x-1} = \frac{9}{4}$

$\left(\frac{2}{3}\right)^{x-1} = \left(\frac{3}{2}\right)^{2}$

$\left(\frac{2}{3}\right)^{x-1} = \left(\frac{2}{3}\right)^{-2}$

$x - 1 = -2$

$x = -1$

b $0{,}2^{x^2} = 5^{-12-4x}$

$\left(\frac{1}{5}\right)^{x^2} = 5^{-12-4x}$

$(5^{-1})^{x^2} = 5^{-12-4x}$

$5^{-x^2} = 5^{-12-4x}$

$-x^2 = -12 - 4x$

$-x^2 + 4x + 12 = 0$

$x^2 - 4x - 12 = 0$

$(x - 6)(x + 2) = 0$

$x = 6 \ \vee \ x = -2$

c $2^{-x} + \left(\frac{1}{2}\right)^{x} = 8$

$2^{-x} + (2^{-1})^{x} = 8$

$2^{-x} + 2^{-x} = 8$

$2 \cdot 2^{-x} = 8$

$2^{-x} = 4$

$2^{-x} = 2^{2}$

$-x = 2$

$x = -2$

d $2^{x+3} - 3 \cdot 2^{x} = 40\sqrt{2}$

$2^{x} \cdot 2^{3} - 3 \cdot 2^{x} = 40\sqrt{2}$

$8 \cdot 2^{x} - 3 \cdot 2^{x} = 40\sqrt{2}$

$5 \cdot 2^{x} = 40\sqrt{2}$

$2^{x} = 8\sqrt{2}$

$2^{x} = 2^{3} \cdot 2^{\frac{1}{2}}$

$2^{x} = 2^{3\frac{1}{2}}$

$x = 3\frac{1}{2}$

2 a $4^{3x-1} = 5$

$3x - 1 = {}^{4}\log(5)$

$3x = 1 + {}^{4}\log(5)$

$x = \frac{1}{3} + \frac{1}{3} \cdot {}^{4}\log(5)$

b $8^{\frac{1}{2}x-4} = 10$

$\frac{1}{2}x - 4 = {}^{8}\log(10)$

$\frac{1}{2}x = 4 + {}^{8}\log(10)$

$x = 8 + 2 \cdot {}^{8}\log(10)$

c $9^{x} - 18 = 7 \cdot 3^{x}$

$(3^{2})^{x} - 7 \cdot 3^{x} - 18 = 0$

$(3^{x})^{2} - 7 \cdot 3^{x} - 18 = 0$

Stel $3^{x} = p$.

$p^2 - 7p - 18 = 0$

$(p - 9)(p + 2) = 0$

$p = 9 \quad \vee \quad p = -2$

$3^{x} = 9 \qquad 3^{x} = -2$

$x = 2 \qquad$ geen oplossing

Dus $x = 2$.

d $3^{x+1} + 3^{2x-1} = 12$

$3^{x} \cdot 3^{1} + 3^{2x} \cdot 3^{-1} = 12$

$3 \cdot 3^{x} + \frac{1}{3} \cdot 3^{2x} = 12$

Stel $3^{x} = p$.

$3p + \frac{1}{3}p^2 = 12$

$9p + p^2 = 36$

$p^2 + 9p - 36 = 0$

$(p + 12)(p - 3) = 0$

$p = -12 \quad \vee \quad p = 3$

$3^{x} = -12 \qquad 3^{x} = 3$

geen oplossing $\qquad x = 1$

Dus $x = 1$.

3 a $f(p) - g(p) = 2 \quad \vee \quad g(p) - f(p) = 2$

$3^{p-1} - 4 - (2 - 3^p) = 2$
$3^p \cdot 3^{-1} - 4 - 2 + 3^p = 2$
$\frac{1}{3} \cdot 3^p + 3^p = 8$
$\frac{4}{3} \cdot 3^p = 8$
$3^p = 6$
$p = {}^3\log(6)$

$2 - 3^p - (3^{p-1} - 4) = 2$
$2 - 3^p - 3^p \cdot 3^{-1} + 4 = 2$
$-3^p - \frac{1}{3} \cdot 3^p = -4$
$-\frac{4}{3} \cdot 3^p = -4$
$3^p = 3$
$p = 1$

Dus $p = {}^3\log(6) \vee p = 1$.

b $f(r) = g(r+1)$ met $q = f(r) \quad \vee \quad g(r) = f(r+1)$ met $q = g(r)$

$3^{r-1} - 4 = 2 - 3^{r+1}$
$3^r \cdot 3^{-1} - 4 = 2 - 3 \cdot 3^r$
$3\frac{1}{3} \cdot 3^r = 6$
$3^r = \dfrac{6}{3\frac{1}{3}} = \dfrac{9}{5}$
$q = 3^{r-1} - 4 = \frac{1}{3} \cdot 3^r - 4 = \frac{1}{3} \cdot \frac{9}{5} - 4 = -3\frac{2}{5}$

$2 - 3^r = 3^{r+1-1} - 4$
$2 - 3^r = 3^r - 4$
$-2 \cdot 3^r = -6$
$3^r = 3$
$q = 2 - 3^r = 2 - 3 = -1$

Dus $q = -3\frac{2}{5} \vee q = -1$.

4 a $^3\log(5) + 2 \cdot {}^3\log(2) - {}^3\log(7) = {}^3\log(5) + {}^3\log(2^2) - {}^3\log(7) = {}^3\log\left(\dfrac{5 \cdot 2^2}{7}\right) = {}^3\log(\frac{20}{7})$

b $3 + {}^2\log(5) = {}^2\log(2^3) + {}^2\log(5) = {}^2\log(2^3 \cdot 5) = {}^2\log(40)$

c $^2\log(8) + {}^3\log(0{,}2) = 3 + {}^3\log(0{,}2) = {}^3\log(3^3) + {}^3\log(0{,}2) = {}^3\log(3^3 \cdot 0{,}2) = {}^3\log(5{,}4)$

d $^2\log(3) - {}^{\frac{1}{2}}\log(5) = {}^2\log(3) + {}^2\log(5) = {}^2\log(3 \cdot 5) = {}^2\log(15)$

5 a $2 \cdot {}^2\log(x-1) = 1 + {}^2\log(18)$
$^2\log((x-1)^2) = {}^2\log(2) + {}^2\log(18)$
$^2\log((x-1)^2) = {}^2\log(2 \cdot 18)$
$(x-1)^2 = 36$
$x - 1 = -6 \quad \vee \quad x - 1 = 6$
$x = -5 \quad \vee \quad x = 7$
voldoet niet $\quad$ voldoet
Dus $x = 7$.

b $^2\log(x) = 3 - {}^2\log(x+2)$
$^2\log(x) + {}^2\log(x+2) = 3$
$^2\log(x(x+2)) = {}^2\log(2^3)$
$x^2 + 2x = 8$
$x^2 + 2x - 8 = 0$
$(x+4)(x-2) = 0$
$x = -4 \quad \vee \quad x = 2$
voldoet niet $\quad$ voldoet
Dus $x = 2$.

6 a $f(a) - g(a) = 1 \quad \vee \quad g(a) - f(a) = 1$

$^2\log(a+3) - {}^2\log(4a) = 1$
$^2\log\left(\dfrac{a+3}{4a}\right) = 1$
$\dfrac{a+3}{4a} = 2$
$a + 3 = 8a$
$-7a = -3$
$a = \frac{3}{7}$
voldoet

$^2\log(4a) - {}^2\log(a+3) = 1$
$^2\log\left(\dfrac{4a}{a+3}\right) = 1$
$\dfrac{4a}{a+3} = 2$
$4a = 2(a+3)$
$4a = 2a + 6$
$2a = 6$
$a = 3$
voldoet

Dus $a = \frac{3}{7} \vee a = 3$.

b $f(p) = g(p+3)$ met $b = f(p)$ $\vee$ $g(p) = f(p+3)$ met $b = g(p)$

$^2\log(p+3) = {}^2\log(4(p+3))$
$p + 3 = 4p + 12$
$-3p = 9$
$p = -3$
voldoet niet

$^2\log(4p) = {}^2\log(p+3+3)$
$^2\log(4p) = {}^2\log(p+6)$
$4p = p + 6$
$3p = 6$
$p = 2$ voldoet
$b = {}^2\log(4p) = {}^2\log(8) = 3$

Dus $b = 3$.

7 a $\dfrac{3e^3 - e^3}{e^2} = \dfrac{2e^3}{e^2} = 2e$

b $(e^{3x} - 3)^2 = (e^{3x})^2 - 2 \cdot e^{3x} \cdot 3 + 3^2 = e^{6x} - 6e^{3x} + 9$

8 a $3x\,e^{2x} - e^{2x} = 0$
$(3x-1)e^{2x} = 0$
$3x - 1 = 0 \vee e^{2x} = 0$
$3x = 1$ geen opl.
$x = \frac{1}{3}$

b $e^{2x-1} - \sqrt[3]{e^2} = 0$
$e^{2x-1} = \sqrt[3]{e^2}$
$e^{2x-1} = e^{\frac{2}{3}}$
$2x - 1 = \frac{2}{3}$
$2x = 1\frac{2}{3}$
$x = \frac{5}{6}$

c $e^{4x} - e^{x+1} = 0$
$e^{4x} = e^{x+1}$
$4x = x + 1$
$3x = 1$
$x = \frac{1}{3}$

d $e^{2x} + 2e^x = 3$
Stel $e^x = p$.
$p^2 + 2p = 3$
$p^2 + 2p - 3 = 0$
$(p+3)(p-1) = 0$
$p = -3 \vee p = 1$
$e^x = -3$ $e^x = 1$
geen opl. $x = 0$

9 a $f(x) = 2e^x - 3x^2$ geeft $f'(x) = 2e^x - 6x$

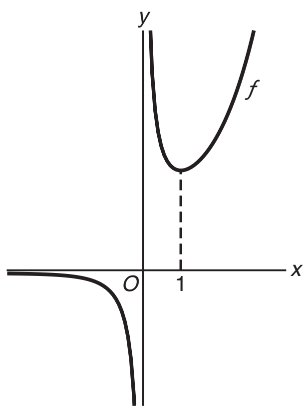
b $g(x) = \dfrac{x^2+1}{e^x}$ geeft $g'(x) = \dfrac{e^x \cdot 2x - (x^2+1) \cdot e^x}{(e^x)^2} = \dfrac{(-x^2+2x-1)e^x}{e^{2x}} = \dfrac{-x^2+2x-1}{e^x}$

c $h(x) = (x^2+1)e^x$ geeft $h'(x) = 2x \cdot e^x + (x^2+1) \cdot e^x = (x^2+2x+1)e^x$

d $j(x) = \dfrac{e^x}{x^2+1}$ geeft $j'(x) = \dfrac{(x^2+1) \cdot e^x - e^x \cdot 2x}{(x^2+1)^2} = \dfrac{(x^2-2x+1)e^x}{(x^2+1)^2}$

10 a $f(x) = \dfrac{e^x}{x}$ geeft $f'(x) = \dfrac{x \cdot e^x - e^x \cdot 1}{x^2} = \dfrac{(x-1)e^x}{x^2}$

$f'(x) = 0$ geeft $\dfrac{(x-1)e^x}{x^2} = 0$
$(x-1)e^x = 0$
$x - 1 = 0 \vee e^x = 0$
$x = 1$ geen opl.



min. is $f(1) = \dfrac{e^1}{1} = e$

b Stel $l: y = ax + b$ met $a = f'(2) = \frac{(2-1)\cdot e^2}{2^2} = \frac{1}{4}e^2$.

$$\left.\begin{array}{l} l: y = \frac{1}{4}e^2 x + b \\ f(2) = \frac{e^2}{2} \text{ dus } A(2, \frac{1}{2}e^2) \end{array}\right\} \begin{array}{l} \frac{1}{2}e^2 = \frac{1}{4}e^2 \cdot 2 + b \\ 0 = b \end{array}$$

Dus $l: y = \frac{1}{4}e^2 x$.

11 a $f(x) = \sqrt{x^2+9} = \sqrt{u}$ met $u = x^2 + 9$ geeft $f'(x) = \frac{dy}{du} \cdot \frac{du}{dx} = \frac{1}{2\sqrt{u}} \cdot 2x = \frac{x}{\sqrt{x^2+9}}$

b $g(x) = e^{x^2+9} = e^u$ met $u = x^2 + 9$ geeft $g'(x) = \frac{dy}{du} \cdot \frac{du}{dx} = e^u \cdot 2x = 2x\, e^{x^2+9}$

c $h(x) = e^x \cdot \sqrt{x^2+9}$ geeft $h'(x) = e^x \cdot \sqrt{x^2+9} + e^x \cdot [\sqrt{x^2+9}]'$

$$= e^x \cdot \sqrt{x^2+9} + e^x \cdot \frac{x}{\sqrt{x^2+9}} = \left(\sqrt{x^2+9} + \frac{x}{\sqrt{x^2+9}}\right)e^x$$

d $j(x) = \frac{e^{3x+1}}{e^{3x}+1}$ geeft

$$j'(x) = \frac{(e^{3x}+1)\cdot 3e^{3x+1} - e^{3x+1}\cdot 3e^{3x}}{(e^{3x}+1)^2} = \frac{3e^{6x+1} + 3e^{3x+1} - 3e^{6x+1}}{(e^{3x}+1)^2} = \frac{3e^{3x+1}}{(e^{3x}+1)^2}$$

12 a $0{,}5 + 3\ln(2) = \ln(e^{\frac{1}{2}}) + \ln(2^3) = \ln(e^{\frac{1}{2}} \cdot 2^3) = \ln(8\sqrt{e})$

b $\ln(e^2) + {}^2\log(5) = 2 + {}^2\log(5) = {}^2\log(2^2) + {}^2\log(5) = {}^2\log(2^2 \cdot 5) = {}^2\log(20)$

13 a $\ln(2-x) - \ln(1-x) = \ln(-x)$

$\ln\left(\frac{2-x}{1-x}\right) = \ln(-x)$

$\frac{2-x}{1-x} = -x$

$2 - x = -x(1-x)$

$2 - x = -x + x^2$

$-x^2 + 2 = 0$

$x^2 = 2$

$x = -\sqrt{2} \quad \vee \quad x = \sqrt{2}$

voldoet — voldoet niet

Dus $x = -\sqrt{2}$.

b $\ln^2(x) - 3\ln(x) + 2 = 0$

Stel $\ln(x) = p$.

$p^2 - 3p + 2 = 0$

$(p-1)(p-2) = 0$

$p = 1 \quad \vee \quad p = 2$

$\ln(x) = 1 \quad \vee \quad \ln(x) = 2$

$x = e \quad \vee \quad x = e^2$

14 a $f(x) = 3^{x-1} + 3^{-x+1}$ geeft $f'(x) = 3^{x-1} \cdot \ln(3) + 3^{-x+1} \cdot \ln(3) \cdot -1 = (3^{x-1} - 3^{-x+1})\ln(3)$

$f'(x) = \frac{8}{3}\ln(3)$ geeft

$(3^{x-1} - 3^{-x+1})\ln(3) = \frac{8}{3}\ln(3)$

$3^{x-1} - 3^{-x+1} = \frac{8}{3}$

$3^x \cdot 3^{-1} - 3^{-x} \cdot 3^1 = \frac{8}{3}$

$\frac{1}{3} \cdot 3^x - 3 \cdot \frac{1}{3^x} = \frac{8}{3}$

Stel $3^x = p$.

$\frac{1}{3}p - 3 \cdot \frac{1}{p} = \frac{8}{3}$ $\quad \times 3p$

$p^2 - 9 = 8p$

$p^2 - 8p - 9 = 0$

$(p-9)(p+1) = 0$

$p = 9 \quad \vee \quad p = -1$

$3^x = 9 \quad \vee \quad 3^x = -1$

$x = 2$ — geen opl.

$f(2) = 3^1 + 3^{-1} = 3\frac{1}{3}$, dus het raakpunt is $(2, 3\frac{1}{3})$.

b $f(p) = f(p+4)$ met $a = f(p)$

$3^{p-1} + 3^{-p+1} = 3^{p+4-1} + 3^{-(p+4)+1}$

$3^{p-1} + 3^{-p+1} = 3^{p+3} + 3^{-p-3}$

$3^p \cdot 3^{-1} + \dfrac{3^1}{3^p} = 3^p \cdot 3^3 + \dfrac{1}{3^{p+3}}$

$\frac{1}{3} \cdot 3^p + \dfrac{3}{3^p} = 27 \cdot 3^p + \dfrac{1}{27 \cdot 3^p}$

Stel $3^p = q$.

$\frac{1}{3}q + \dfrac{3}{q} = 27q + \dfrac{1}{27q}$

$\left(3 - \frac{1}{27}\right) \cdot \frac{1}{q} = \left(27 - \frac{1}{3}\right)q$

$\dfrac{80}{27q} = \dfrac{80q}{3}$

$80 \cdot 27q^2 = 3 \cdot 80$

$q^2 = \frac{1}{9}$

$q = -\frac{1}{3} \quad \vee \quad q = \frac{1}{3}$

$3^p = -\frac{1}{3} \quad \vee \quad 3^p = \frac{1}{3}$

geen opl. $\qquad 3^p = 3^{-1}$

$p = -1$

$a = f(-1) = 3^{-2} + 3^2 = 9\frac{1}{9}$

15 a $f(x) = {}^3\log(5x-6) = {}^3\log(u)$ met $u = 5x - 6$ geeft

$f'(x) = \dfrac{dy}{du} \cdot \dfrac{du}{dx} = \dfrac{1}{u\ln(3)} \cdot 5 = \dfrac{5}{(5x-6)\ln(3)}$

b $g(x) = \ln(3x^2+3) = \ln(u)$ met $u = 3x^2 + 3$ geeft $g'(x) = \dfrac{dy}{du} \cdot \dfrac{du}{dx} = \dfrac{1}{u} \cdot 6x = \dfrac{6x}{3x^2+3} = \dfrac{2x}{x^2+1}$

c $h(x) = \dfrac{7^x}{{}^7\log(x)}$ geeft $h'(x) = \dfrac{{}^7\log(x) \cdot 7^x \cdot \ln(7) - 7^x \cdot \dfrac{1}{x\ln(7)}}{\left({}^7\log(x)\right)^2}$

d $j(x) = \ln(\ln(x^2)) = \ln(u)$ met $u = \ln(x^2) = \ln(v)$ met $v = x^2$ geeft

$j'(x) = \dfrac{dy}{du} \cdot \dfrac{du}{dv} \cdot \dfrac{dv}{dx} = \dfrac{1}{u} \cdot \dfrac{1}{v} \cdot 2x = \dfrac{2x}{\ln(x^2) \cdot x^2} = \dfrac{2}{x\ln(x^2)}$

16 a $f(x) = \ln(x)$ geeft $f'(x) = \dfrac{1}{x}$

$g(x) = \dfrac{\ln(x)}{x}$ geeft $g'(x) = \dfrac{x \cdot \dfrac{1}{x} - \ln(x) \cdot 1}{x^2} = \dfrac{1 - \ln(x)}{x^2}$

$$\left.\begin{array}{l} f'(1) = \frac{1}{1} = 1 \\ g'(1) = \dfrac{1-\ln(1)}{1^2} = \dfrac{1-0}{1} = 1 \end{array}\right\} f'(1) = g'(1)$$

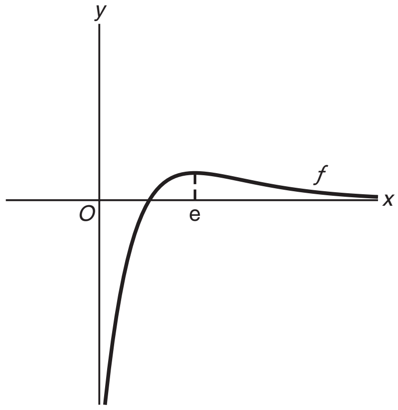
Dus de grafieken van f en g raken elkaar in A.

b $g'(x) = 0$ geeft $\dfrac{1-\ln(x)}{x^2} = 0$

$1 - \ln(x) = 0$

$\ln(x) = 1$

$x = e$



$g(e) = \dfrac{\ln(e)}{e} = \dfrac{1}{e}$

$B_g = \left\langle \leftarrow, \dfrac{1}{e}\right]$