Diagnostische toets

bladzijde 160

1 a beginterm $u_0 = 7$, telkens $+3$ } $u_n = 7 + 3n$

20^e term is $u_{19} = 7 + 3 \cdot 19 = 64$

b beginterm $u_0 = 3$, telkens $\times 3$ } $u_n = 3 \cdot 3^n$

20^e term is $u_{19} = 3 \cdot 3^{19} = 3\,486\,784\,401$

c beginterm $u_0 = -3$, telkens $\times -1$ } $u_n = -3 \cdot (-1)^n$

20^e term is $u_{19} = -3 \cdot (-1)^{19} = 3$

d $3 = 1^2 + 2, 6 = 2^2 + 2, 11 = 3^2 + 2, 18 = 4^2 + 2$, enz.

Dus $u_n = (n+1)^2 + 2$.

20^e term is $u_{19} = (19+2)^2 + 2 = 402$

2 a Voer in 100 en $5 + 20\sqrt{\text{ANS}}$.

Je krijgt $u_6 \approx 402$ en $u_9 \approx 409$

b $u_{13} \approx 409{,}88$ en $u_{14} \approx 409{,}91$

Vanaf de 15^e term is $u_n > 409{,}9$.

c Ga door met op ENTER te drukken.

Je krijgt grenswaarde $\approx 409{,}939$.

3 a Voer de formule in op het rijen-invoerscherm.

Je krijgt $u_0 = 1, u_1 = 4, u_2 = 20, u_3 = 87, u_4 = 325$ en $u_5 = 1100$.

b Je krijgt $u_{11} = 906\,887$ en $u_{12} = 2\,722\,389$.

Dus vanaf $n = 12$ is $u_n > 1\,000\,000$.

4 a rr met $b = 130$ en $v = -5$, dus $u_n = 130 - 5n$

b 20^e term is $u_{19} = 130 - 5 \cdot 19 = 35$

5 a rr met $b = u_0 = 25$ en $v = 7$, dus

recursieve formule $u_n = u_{n-1} + 7$ met $u_0 = 25$

directe formule $u_n = 25 + 7n$

b Los op $25 + 7n = 130$

$7n = 105$

$n = 15$

De 16^e term is 130.

6 a $u_0 = -5$ en $u_{25} = 8 \cdot 25 - 5 = 195$
Dus $S_{25} = \frac{1}{2} \cdot 26 \cdot (-5 + 195) = 2470$.
b De eerste term is $u_0 = -5$.
De 30^e^ term is $u_{29} = 8 \cdot 29 - 5 = 227$.
De som van de eerste 30 termen is $S_{29} = \frac{1}{2} \cdot 30 \cdot (-5 + 227) = 3330$.

7 a rr met $b = 18$ en $v = 12$, dus $u_n = 18 + 12n$
Uit $u_n = 150$ volgt $18 + 12n = 150$
$12n = 132$
$n = 11$
Dus $18 + 30 + 42 + 54 + \ldots + 150 = S_{11} = \frac{1}{2} \cdot (11 + 1) \cdot (18 + 150) = 1008$
b rr met $b = 180$ en $v = -8$, dus $u_n = 180 - 8n$
Uit $u_n = 100$ volgt $180 - 8n = 100$
$-8n = -80$
$n = 10$
Dus $180 + 172 + 164 + 156 + \ldots + 100 = S_{10} = \frac{1}{2} \cdot (10 + 1) \cdot (180 + 100) = 1540$

8 a mr met $b = 100$ en $r = 1{,}18$, dus $u_n = 100 \cdot 1{,}18^n$
b De 10^e^ term is $u_9 = 100 \cdot 1{,}18^9 \approx 443{,}55$.
c Voer in 100 en ANS × 1,18.
Je krijgt $u_{13} \approx 860$ en $u_{14} \approx 1015$.
Dus vanaf $n = 14$ is $u_n > 1000$.

9 a mr met $b = 800$ en $r = 1{,}25$, dus
recursieve formule $u_n = 1{,}25 \cdot u_{n-1}$ met $u_0 = 800$
directe formule $u_n = 800 \cdot 1{,}25^n$
b Voer in 800 en ANS × 1,25.
Je krijgt $u_{14} \approx 18\,190$ en $u_{15} \approx 22\,737$.
Dus vanaf de 16^e^ term is $u_n > 20\,000$.

10 $r^3 = \frac{u_8}{u_5} = \frac{32\,805}{1215} = 27$, dus $r = 3$

$$\left.\begin{array}{l} \text{Dit geeft } u_n = b \cdot 3^n \\ u_5 = 1215 \end{array}\right\} 1215 = b \cdot 3^5$$
$$\frac{1215}{3^5} = b$$
$$b = 5$$
Dus $u_n = 5 \cdot 3^n$.

bladzijde 161

11 a $S_{10} = \frac{b(1 - r^{11})}{1 - r} = \frac{100(1 - 1{,}08^{11})}{1 - 1{,}08} \approx 1664{,}55$
b De som van de eerste vijftien termen is $S_{14} = \frac{b(1 - r^{15})}{1 - r} = \frac{100(1 - 1{,}08^{15})}{1 - 1{,}08} \approx 2715{,}21$.

12 a mr met $u_0 = 5$ en $r = 4$
$u_n = 1\,310\,720$, dus $u_{n+1} = 4 \cdot 1\,310\,720 = 5\,242\,880$
$5 + 20 + 80 + 320 + \ldots + 1\,310\,720 = \frac{u_0 - u_{n+1}}{1 - r} = \frac{5 - 5\,242\,880}{1 - 4} = 1\,747\,625$
b mr met $u_0 = 1{,}15$ en $r = 1{,}15$
$u_n = 1{,}15^{20}$, dus $u_{n+1} = 1{,}15^{21}$
$1{,}15 + 1{,}15^2 + 1{,}15^3 + \ldots + 1{,}15^{20} = \frac{u_0 - u_{n+1}}{1 - r} = \frac{1{,}15 - 1{,}15^{21}}{1 - 1{,}15} \approx 117{,}810$

13 a sommeerbare mr met $b = 1000$ en $r = 0{,}8$

$$S = \frac{b}{1-r} = \frac{1000}{1-0{,}8} = 5000$$

b sommeerbare mr met $b = 100$ en $r = -0{,}6$

$$S = \frac{b}{1-r} = \frac{100}{1--0{,}6} = \frac{100}{1{,}6} = 62{,}5$$

c sommeerbare mr met $b = 50$ en $r = 0{,}95$

$$S = \frac{b}{1-r} = \frac{50}{1-0{,}95} = 1000$$

d sommeerbare mr met $b = 600$ en $r = -0{,}25$

$$S = \frac{b}{1-r} = \frac{600}{1--0{,}25} = \frac{600}{1{,}25} = 480$$

14 a u_n is een rr met $b = 10$ en $v = 8$, dus $u_n = 10 + 8n$

$S_n = S_{n-1} + u_n$ met $S_0 = u_0$ geeft $S_n = S_{n-1} + 10 + 8n$ met $S_0 = 10$

b u_n is een mr met $b = 10$ en $r = 1{,}8$, dus $u_n = 10 \cdot 1{,}8^n$

$S_n = S_{n-1} + u_n$ met $S_0 = u_0$ geeft $S_n = S_{n-1} + 10 \cdot 1{,}8^n$ met $S_0 = 10$

15 a TI-83

Voer in $u_n = 0{,}5n^2 + 4n + 3$
$v_n = v_{n-1} + u$
$v_0 = 3$

$S_{15} = v_{15} = 1148$

Casio

Voer in $a_n = 0{,}5n^2 + 4n + 3$
en zet Σ Display op On.

$S_{15} = \Sigma a_{15} = 1148$

b TI-83

Voer in $u_n = 0{,}5u_{n-1} + 2n$
$u_0 = 10$
$v_n = v_{n-1} + u$
$v_0 = 10$

$S_{15} = v_{15} \approx 444{,}00$

Casio

Voer in $a_{n+1} = 0{,}5a_n + 2(n+1)$
met $a_0 = 10$
en zet Σ Display op On.

$S_{15} = \Sigma a_{15} \approx 444{,}00$

16 a $\displaystyle\sum_{n=3}^{7}(n^2+1) = (3^2+1) + (4^2+1) + (5^2+1) + (6^2+1) + (7^2+1) =$

$10 + 17 + 26 + 37 + 50 = 140$

b $\displaystyle\sum_{n=10}^{15}(n^2-n) = (10^2-10) + (11^2-11) + (12^2-12) + (13^2-13) +$

$(14^2-14) + (15^2-15) = 90 + 110 + 132 + 156 + 182 + 210 = 880$

17 a $\displaystyle\sum_{n=0}^{20}(50 \cdot 1{,}1^n) = S_{20} = \frac{b(1-r^{21})}{1-r} = \frac{50(1-1{,}1^{21})}{1-1{,}1} \approx 3200{,}12$

b TI-83

Voer in $n\text{Min} = 5$
$u_n = \dfrac{100}{2n+1}$
$v_n = v_{n-1} + u$
$v_5 = \dfrac{100}{2 \cdot 5 + 1}$

Je krijgt $v_{25} \approx 82{,}35$,
dus $\displaystyle\sum_{n=5}^{25}\frac{100}{2n+1} \approx 82{,}35$.

Casio

Voer in $a_n = \dfrac{100}{2n+1}$, zet
Σ Display op On en
Table Start op 5.

Je krijgt $\Sigma a_{25} \approx 82{,}35$,
dus $\displaystyle\sum_{n=5}^{25}\frac{100}{2n+1} \approx 82{,}35$.

c $\sum_{n=10}^{16}(50 \cdot 0{,}95^n) = S_{16} - S_9 = \frac{50(1-0{,}95^{17})}{1-0{,}95} - \frac{50(1-0{,}95^{10})}{1-0{,}95} \approx 581{,}880 - 401{,}263 \approx 180{,}62$

d TI-83

Voer in nMin = 3

$u_n = 50 \cdot \frac{6n}{n+3}$

$v_n = v_{n-1} + u$

$v_3 = 50 \cdot \frac{6 \cdot 3}{3+3}$

Je krijgt $v_{18} \approx 3574{,}18$,

dus $\sum_{n=3}^{18} 50 \cdot \frac{6n}{n+3} \approx 3574{,}18$

Casio

Voer in $a_n = 50 \cdot \frac{6n}{n+3}$, zet

Σ Display op On en

Table Start op 3.

Je krijgt $\Sigma a_{18} \approx 3574{,}18$,

dus $\sum_{n=3}^{18} 50 \cdot \frac{6n}{n+3} \approx 3574{,}18$.

18 a $\Delta u_n = u_{n+1} - u_n = 3(n+1)^2 + (n+1) - (3n^2 + n)$

$= 3(n^2 + 2n + 1) + n + 1 - 3n^2 - n$

$= 3n^2 + 6n + 3 + n + 1 - 3n^2 - n$

$= 6n + 4$

b $\Delta u_n = u_{n+1} - u_n = 600 \cdot 1{,}15^{n+1} - 600 \cdot 1{,}15^n$

$= 600 \cdot 1{,}15^n(1{,}15 - 1)$

$= 600 \cdot 1{,}15^n \cdot 0{,}15$

$= 90 \cdot 1{,}15^n$