Diagnostische toets

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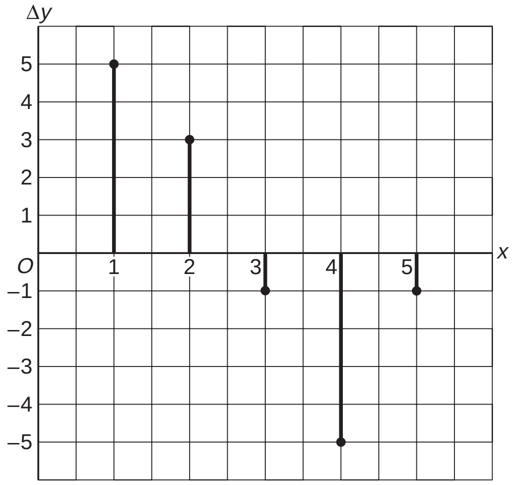
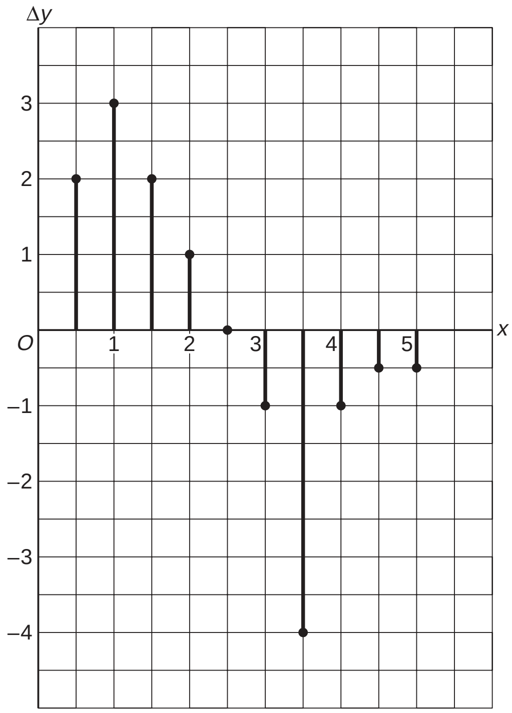
1

x	y	Δy
0	1	–
1	6	5
2	9	3
3	8	–1
4	3	–5
5	2	–1



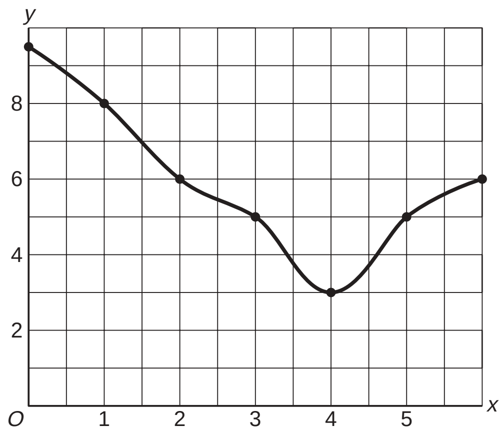
b

x	y	Δy
0	1	–
0,5	3	2
1	6	3
1,5	8	2
2	9	1
2,5	9	0
3	8	–1
3,5	4	–4
4	3	–1
4,5	2,5	–0,5
5	2	–0,5



2 Door (3, 5) en op [3, 4] is $\Delta y = -2$, dus ook door (4, 3).
Door (3, 5) en op [2, 3] is $\Delta y = -1$, dus ook door (2, 6).
Enzovoort, zie de tabel.

x	0	1	2	3	4	5	6
y	9,5	8	6	5	3	5	6



3 a Op [0, 2] is $\dfrac{\Delta y}{\Delta x} = \dfrac{9-1}{2-0} = \dfrac{8}{2} = 4$.

Op [2, 5] is $\dfrac{\Delta y}{\Delta x} = \dfrac{2-9}{5-2} = \dfrac{-7}{3} = -2\frac{1}{3}$.

b Op [1, 3] is $\dfrac{\Delta y}{\Delta x} = \dfrac{8-6}{3-1} = \dfrac{2}{2} = 1$.

Op [2, 4] is $\dfrac{\Delta y}{\Delta x} = \dfrac{3-9}{4-2} = \dfrac{-6}{2} = -3$.

4 a Op [10, 30] is de gemiddelde snelheid $\dfrac{\Delta s}{\Delta t} = \dfrac{30-7}{30-10} = \dfrac{23}{20} = 1{,}15$ km per minuut $= 69$ km/uur.

b Teken de lijn door (0, 0) en (7,5; 6) door totdat je de grafiek weer snijdt. Dat is in het punt (17,5; 14).
Dus $t = 17{,}5$.

5 a $\dfrac{\Delta y}{\Delta x} = \dfrac{f(4)-f(1)}{4-1} = \dfrac{8 - -2{,}5}{3} = \dfrac{10{,}5}{3} = 3{,}5$

b $\dfrac{\Delta y}{\Delta x} = \dfrac{f(1)-f(-1)}{1 - -1} = \dfrac{-2{,}5-0{,}5}{2} = \dfrac{-3}{2} = -1{,}5$

c $\left.\begin{array}{l} f(-2) = -4,\text{ dus } A(-2,\ -4) \\ f(3) = -1{,}5,\text{ dus } B(3; -1{,}5) \end{array}\right\}$ helling $= \dfrac{\Delta y}{\Delta x} = \dfrac{-1{,}5 - -4}{3 - -2} = 0{,}5$

6 a Voer in $y_1 = \sqrt{2x-3}$.

Helling $= \left[\dfrac{dy}{dx}\right]_{x=2} = 1$.

b Snelheid $= \left[\dfrac{dy}{dx}\right]_{x=3{,}5} = 0{,}5$.

c $k\colon y = ax + b$ met $a = \left[\frac{dy}{dx}\right]_{x=6} = \frac{1}{3}$.
Dus $k\colon y = \frac{1}{3}x + b$
$\left.\begin{array}{l} \\ y_B = f(6) = 3, \text{ dus } B(6,\ 3)\end{array}\right\}\ \begin{array}{l} 3 = \frac{1}{3}\cdot 6 + b \\ 3 = 2 + b \\ 1 = b \end{array}$

Dus $k\colon y = \frac{1}{3}x + 1$.

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7 a Voer in $y_1 = -0{,}25x^3 + 3x^2$.

De snelheid op $t = 2$ is $\left[\frac{ds}{dt}\right]_{t=2} = 9$ m/s.

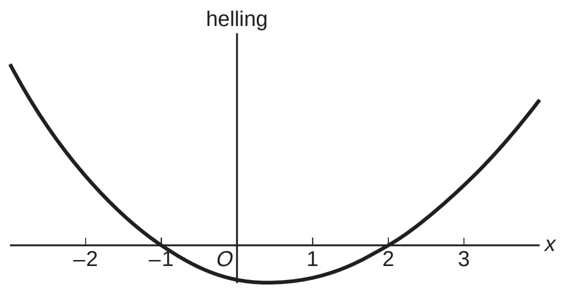
De snelheid op $t = 4$ is $\left[\frac{ds}{dt}\right]_{t=4} = 12$ m/s.

b Na 6 seconden is afgelegd $s = -0{,}25 \cdot 6^3 + 3 \cdot 6^2 = 54$ m.

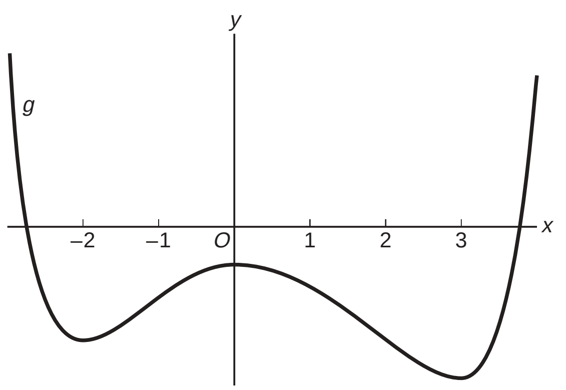
De snelheid op $t = 6$ is $\left[\frac{ds}{dt}\right]_{t=6} = 9$ m/s.

Tussen $t = 6$ en $t = 10$ wordt $4 \cdot 9 = 36$ m afgelegd.
Dus na 10 seconden is $54 + 36 = 90$ m afgelegd.

8



9



10 a $f(x) = 0{,}6x^3 - 1{,}3x^2 + 7$ geeft $f'(x) = 1{,}8x^2 - 2{,}6x$
b $g(p) = 4p^3 + p^2 - 11p + 20$ geeft $g'(p) = 12p^2 + 2p - 11$
c $h(q) = 3q - 2(q^2 - 4q) = 3q - 2q^2 + 8q = -2q^2 + 11q$ geeft $h'(q) = -4q + 11$
d $k(x) = ax^2 + bx + c$ geeft $k'(x) = 2ax + b$

11 a $f(x) = (3 - x)(5 + 2x) = 15 + 6x - 5x - 2x^2 = -2x^2 + x + 15$ geeft $f'(x) = -4x + 1$

b $g(x) = (3x + 1)^2 = (3x + 1)(3x + 1) = 9x^2 + 3x + 3x + 1 = 9x^2 + 6x + 1$
geeft $g'(x) = 18x + 6$

c $h(x) = x(2x - 1)^2 = x(2x - 1)(2x - 1) = x(4x^2 - 2x - 2x + 1)$
$= x(4x^2 - 4x + 1) = 4x^3 - 4x^2 + x$ geeft $h'(x) = 12x^2 - 8x + 1$

d $k(x) = \frac{1}{3}x^3 + 2x^2(x - 4) + 6 = \frac{1}{3}x^3 + 2x^3 - 8x^2 + 6 =$
$2\frac{1}{3}x^3 - 8x^2 + 6$
geeft $k'(x) = 7x^2 - 16x$

12 a $f(x) = 0{,}2x^3 - 6x + 2$ geeft $f'(x) = 0{,}6x^2 - 6$
Stel $m\colon y = ax + b$ met $a = f'(5) = 0{,}6 \cdot 5^2 - 6 = 9$.
Dus $m\colon y = 9x + b$.
$y_A = f(5) = -3$, dus $A(5, -3)$
Invullen geeft $-3 = 9 \cdot 5 + b$
$-3 = 45 + b$
$-48 = b$
Dus $m\colon y = 9x - 48$.

b B is het snijpunt met de y-as, dus $x_B = 0$.
Stel $k\colon y = ax + b$ met $a = f'(0) = 0{,}6 \cdot 0^2 - 6 = -6$.
Dus $k\colon y = -6x + b$.
$y_B = f(0) = 2$, dus $B(0, 2)$
Dit geeft $k\colon y = -6x + 2$.

13 a $f(x) = -\frac{1}{6}x^3 + \frac{1}{2}x^2 + 4x + 1$ geeft $f'(x) = -\frac{1}{2}x^2 + x + 4$
Stel $k\colon y = ax + b$ met $a = f'(2) = -\frac{1}{2} \cdot 2^2 + 2 + 4 = 4$.
Dus $k\colon y = 4x + b$.
$y_A = f(2) = 9\frac{2}{3}$, dus $A(2, 9\frac{2}{3})$
Invullen geeft $9\frac{2}{3} = 4 \cdot 2 + b$
$9\frac{2}{3} = 8 + b$
$1\frac{2}{3} = b$
Dus $k\colon y = 4x + 1\frac{2}{3}$.

b m is evenwijdig met k, dus $\text{rc}_m = \text{rc}_k = 4$, dus $f'(x) = 4$.
$-\frac{1}{2}x^2 + x + 4 = 4$
$-\frac{1}{2}x^2 + x = 0$
$x^2 - 2x = 0$
$x(x - 2) = 0$
$x = 0 \vee x = 2$
Dus $x_B = 0$.
$y_B = f(0) = 1$, dus $B(0, 1)$.

c Raaklijn horizontaal, dus rc $= 0$, dus $f'(x) = 0$.
$-\frac{1}{2}x^2 + x + 4 = 0$
$x^2 - 2x - 8 = 0$
$(x + 2)(x - 4) = 0$
$x = -2 \vee x = 4$
$f(-2) = -3\frac{2}{3}$ en $f(4) = 14\frac{1}{3}$.
De punten zijn $(-2, -3\frac{2}{3})$ en $(4, 14\frac{1}{3})$.