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UITWERKINGEN VOOR HET VWO
A1B1 DEEL1

Hoofdstuk 8

RIJEN

KERN 1
DISCRETE ANALYSE

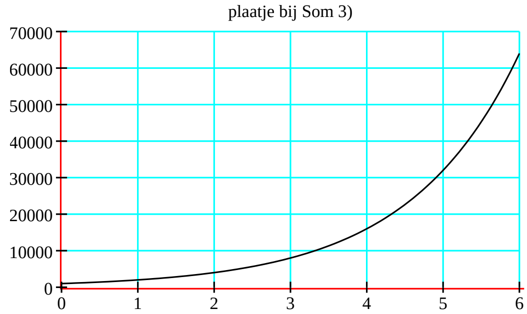
1) II: bij de $1^{ste} grafiek$

III: bij de $2^{de} grafiek$

2) I en **III**

3a) $C = 1000 \cdot 2^r$

3b)

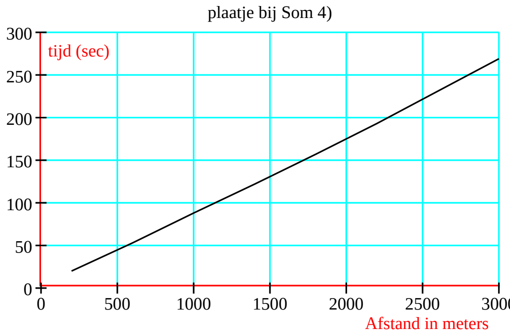


4a)

4b) ja

4c) 2 min. 3 kwartier

4d) neen



5a)

$tijd\ in\ uren$	0	3	6	9	12	15	18	21	24
$Temperatuur\ ^oC$	12	10	10	12	18	20	20	16	13

5b) Volgens de tabel: $\begin{cases} T_{min} = 10^pC \\ T_{max} = 20^oC \end{cases}$

5c) Volgens de grafiek: $\begin{cases} T_{min} = 9^pC \\ T_{max} = 22^oC \end{cases}$

5d) De temperatuur schommelt tussen $10^oC\, en\, 20^oC$, waarbij het 's middags warmer is. Dit klopt redelijk.

5e) De tabel is overzichtelijker: in de Tabel kun je de gegevens direct aflezen
De tabel is onnauwkeuriger dan de grafiek.

5f)

$Voordeel \longrightarrow Nauwkeuriger$

$Nadeel \longrightarrow tabel\ wordt\ erg\ groot$

[1] *Deze samenvatting mag niet massaal op kosten van Schaersvoorde worden Uitgeprint!!!*

[2] *werd gemaakt onder LinuX met* LaTeX *en* LyX

[3] *Typ&andere fouten&blunders graag Melden!*

KERN 2
RIJEN

6a)

$Afstand\,(km)$	5	10	15	20	25	30	35	40
$tijd\,(minuten)$	18	36	54	72	90	108	126	144

6b) $\frac{40}{5} = 8 \quad 8 \cdot 18 = 144$

$\frac{42{,}195}{5} \cdot 18 \simeq 151{,}9 \simeq 152\,minuten$

7a)

1	4	7	10	13	16	19	$\hookleftarrow (+3)$

7b)

1	3	9	27	81	243	729	$\hookleftarrow (\times 3)$

7c)

1	4	9	16	25	36	49	64	$\hookleftarrow (1^2, 2^2, 3^2, 4^2, etc)$

7d)

$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$	$\frac{1}{32}$	$\frac{1}{64}$	$\frac{1}{128}$	$\hookleftarrow \left(\frac{1}{2^1}\right), \left(\frac{1}{2^2}\right), \left(\frac{1}{2^3}\right), \left(\frac{1}{2^4}\right), \left(\frac{1}{2^5}\right), etc$

7e)

1	$\xrightarrow{+3^1}$	4	$\xrightarrow{+3^2}$	13	$\xrightarrow{+3^3}$	40	$\xrightarrow{+3^4}$	121	$\xrightarrow{+3^5}$	364	$\xrightarrow{+3^6}$	1093

7f)

625	125	25	5	1	$\frac{1}{5}$
5^4	5^3	5^2	5^1	5^0	5^{-1}

8a)

-1	1	3	5	7	9	$\longrightarrow t(n) = 2n - 1$

8b)

2	7	12	17	22	27	$\longrightarrow t(n) = 5n + 2$

8c)

1	2	4	8	16	32	$\longrightarrow t(n) = 2^n$

8d)

0	2	6	12	20	30	$\longrightarrow t(n) = n^2 + n$

8e)

0	1	8	27	64	125	$\longrightarrow t(n) = n^3$

8f)

35	35	35	35	35	35	$\longrightarrow t(n) = 35$

9a)

0	$\xrightarrow{+3}$	3	$\xrightarrow{+3}$	6	$\xrightarrow{+3}$	9	$\xrightarrow{+3}$	12	$\xrightarrow{+3}$	15	$\longrightarrow t(n) = 3 \cdot n$

9b)

5	$\xrightarrow{+3}$	8	$\xrightarrow{+3}$	11	$\xrightarrow{+3}$	14	$\xrightarrow{+3}$	17	$\xrightarrow{+3}$	20	$\longrightarrow t(n) = 3 \cdot n + 5$

9c)

1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\dashrightarrow$	$\frac{1}{n}$	
$\frac{1}{2^0}$	$\frac{1}{2^1}$	$\frac{1}{2^2}$	$\frac{1}{2^3}$	$\dashrightarrow$	$\frac{1}{2^n}$	$\longrightarrow t(n) = \frac{1}{2^n}$

9d)

20	$\xrightarrow{\times 2}$	40	$\xrightarrow{\times 2}$	80	$\xrightarrow{\times 2}$	160	$\longrightarrow t(n) = 20 \cdot 2^n$

10a)

Jaar	1	2	3	4
Bedrag Jeroen	10,-	11,10	12,20	13,30
Bedrag Marco	10,-	11,-	12,10	13,31

10b) $Jeroen(n) = 10 + 1,1 \cdot n \quad Marco(n) = 10 \cdot 1,1^{(n-1)}$

11) $Groei \xrightarrow{2{,}3\%\ per\ Jaar} g = 1.023$

11a) $120 \times 1,023 = 122,76 \simeq 122,8\, miljoen$

11b)

$Jaar$	1995	1996	1997	1998	1999	2000
$Aantal$	120	122,8	125,6	128,5	131,4	134,4

11c) $N(t+1) = 1,023 \cdot N(t)$

12a) $t(0) = 18 \quad t(n+1) = t(n) + 4$
$t(0+1) = t(0) + 4 = 18 + 4 = 22$
$t(1+1) = t(1) + 4 = 22 + 4 = 26$
$t(2+1) = t(2) + 4 = 26 + 4 = 30$
$t(3+1) = t(3) + 4 = 30 + 4 = 34$

12b) $t(0) = 160$
$t(1) = t(0+1) = 0,5 \cdot t(0) = 0,5 \cdot 160 = 80$
$t(2) = t(1+1) = 0,5 \cdot t(1) = 0,5 \cdot 80 = 40$
$t(3) = t(2+1) = 0,5 \cdot t(2) = 0,5 \cdot 40 = 20$
$t(4) = t(3+1) = 0,5 \cdot t(3) = 0,5 \cdot 20 = 10$

12c) $t(0) = 1$
$t(1) = t(0+1) = 1 \cdot t(0) = 1 \cdot 1 = 1$
$t(2) = t(1+1) = 2 \cdot t(1) = 2 \cdot 1 = 2$
$t(3) = t(2+1) = 3 \cdot t(2) = 3 \cdot 2 = 6$
$t(4) = t(3+1) = 4 \cdot t(3) = 4 \cdot 6 = 24$

12d) $t(0) = 2$
$t(1) = t(0+1) = \frac{t(0)}{2} + 1 = \frac{2}{2} + 1 = 2$
$t(2) = t(1+1) = \frac{t(1)}{2} + 1 = \frac{2}{2} + 1 = 2$
$t(3) = 2$
$t(4) = 2$

13A
$t(0) = 5$
$t(1) = t \cdot (0+1) = t(0) + 3 = 5 + 3 = 8$
$t(2) = t \cdot (1+1) = t(1) + 3 = 8 + 3 = 11$

n	0		1		2
$t(n)$	5	$\xrightarrow{+3}$	8	$\xrightarrow{+3}$	11

$\xrightarrow{Dus}$**A** hoort bij **I**

13B
$t(0) = 5$
$t(1) = t(0+1) = 3 \cdot t(0) = 3 \cdot 5 = 15$
$t(2) = t(1+1) = 3 \cdot t(1) = 3 \cdot 15 = 3 \cdot 3 \cdot 5 = 45$
$t(3) = t(2+1) = 3 \cdot t(2) = 3 \cdot 45 =$
$= 3 \cdot 3 \cdot 15 = 3 \cdot 3 \cdot 3 \cdot 5 = 135$
$\xrightarrow{Dus}$**B** hoort bij **II**

13C
$t(0) = 3$
$t(1) = t(0+1) = t(0) + 5 = 3 + 5 = 8$
$t(2) = t(1+1) = t(1) + 5 = 8 + 5 = 13$
$\xrightarrow{Dus}$**C** hoort bij **IV**

13D
$t(0) = 3$
$t(1) = t(0+1) = 5 \cdot t(0) = 5 \cdot 3 = 15$
$t(2) = t(1+1) = 5 \cdot t(1) = 5 \cdot 15 = 5 \cdot 5 \cdot 3 = 75$
$\xrightarrow{Dus}$**D** hoort bij **III**

14a)
$t(0) = 4$
$t(1) = 9 \dashrightarrow t(1) = t(0+1) = t(0) + 5$
$t(2) = 14 \dashrightarrow t(2) = t(1+1) = t(1) + 5$
$t(3) = 19 \dashrightarrow t(3) = t(2+1) = t(2) + 5$
$t(4) = 24 \dashrightarrow t(4) = t(3+1) = t(2) + 5$
$\downdownarrows \quad \downdownarrows \quad \downdownarrows$
$t(n) = 5n + 4 \dashrightarrow t(n+1) = t(n) + 5 \; en \; t(0) = 4$
Of: $t(n) = 5n + 4 \longrightarrow$
$\Rightarrow \; t(n+1) = 5(n+1) + 4 \; \Rightarrow$
$\Rightarrow \; t(n+1) = 5n + 4 + 5 \; \Rightarrow$
$\Rightarrow \; t(n+1) = t(n) + 5 \; en \; t(0) = 4$

14b)
$t(0) = 4$
$t(1) = 20 \dashrightarrow t(1) = t(0+1) = 5 \cdot t(0)$
$t(2) = 100 \dashrightarrow t(2) = t(1+1) = 5 \cdot t(1)$
$t(3) = 500 \dashrightarrow t(3) = t(2+1) = 5 \cdot t(2)$
$t(4) = 2500 \dashrightarrow t(4) = t(3+1) = 5 \cdot t(3)$
$\downdownarrows \quad \downdownarrows \quad \downdownarrows$
$t(n) = 4 \cdot 5^n \dashrightarrow t(n+1) = 5 \cdot t(n) \; en \; t(0) = 4$
Of: $t(n) = 4 \cdot 5^n \longrightarrow t(n+1) = 4 \cdot 5^{(n+1)} \; \Rightarrow$
$t(n+1) = 5 \cdot 4 \cdot 5^n \; \Rightarrow$
$\Rightarrow \; t(n+1) = 5 \cdot t(n) \; en \; t(0) = 4$

15)
$t(0) = 1$
$t(1) = 1$
$t(2) = t(1+1) = t(1) + t(0) = 1 + 1 = 2$
$t(3) = t(2+1) = t(2) + t(1) = 2 + 1 = 3$
$t(4) = t(3+1) = t(3) + t(2) = 3 + 2 = 5$
$t(5) = t(4+1) = t(4) + t(3) = 5 + 3 = 8$
$t(6) = t(5+1) = t(5) + t(4) = 8 + 5 = 13$
$t(7) = t(6+1) = t(6) + t(5) = 13 + 8 = 21$
$t(8) = t(7+1) = t(7) + t(6) = 21 + 13 = 34$
$t(9) = t(8+1) = t(8) + t(7) = 34 + 21 = 55$
$t(n+1) = t(n) + t(n-1)$

16a) Na 1 maand (1 februari.) is er nog steeds 1 paar
Na 2 maand (1 maart) zijn er twee paren (1^{ste}paar 1 keer gejongd)

16b)

$Maand$	$Aantal\,Paren\,Jongen$	$Totaal\,Aantal\,Paren$	$\triangle$
1 jan	1	1	$(1-1)=0$ paren die jongen
1 feb	0	1	$(1-0)=1$ paar welke jongt
1 mrt	1	2	$(2-1)=1$ paar welke jongt
1 april	1	3	$(3-1)=2$ paren die jongen
1 mei	2	5	$(5-2)=3$ paren die jongen
1 juni	3	8	$(8-3)=5$ paren de jongen
1 juli	5	13	enz
1 sept	138	21	enz
1 okt	21	55	
1 nov	34	89	
1 dec	55	144	
1 jan	89	233	

16b) Dus 233 Konijnen

17a) $t(0)=5 \longrightarrow is\ oneven$

$t(1)=t(0+1)=3\cdot t(0)+1=16 \xleftarrow{is\,Even}$

$t(2)=t(1+1)=\frac{t(1)}{2}=8 \xleftarrow{is\,Even}$

$t(3)=t(3+1)=\frac{t(2)}{2}=4 \xleftarrow{is\,Even}$

$t(4)=t(3+1)=\frac{t(3)}{2}=2 \xleftarrow{is\,Even}$

$t(5)=t(4+1)=\frac{t(4)}{2}=1 \xleftarrow{is\,Onven}$

17b)

$t(0)=6 \longrightarrow 6,3,10,5,16,8,4,2,1,4,2,1,4,2,1$

$t(0)=4 \longrightarrow 4,2,1,4,2,1$

$t(0)=47 \longrightarrow 47,142,71,214,107,322,161,484,242,121,364,182,91,274,137,412,206,$
103,310,155,466,233,700,350,175,526,263,790,395,1186,593,1780,890,445,1330,668,334,167,502,251
754,277,1132,566,283...........$\rightsquigarrow$misschien op de hele lange duur wel, maar nu even niet.........

17c) $t(0)=111 \longrightarrow 111,334,167,502,251,754,377,1132$

17d) $t(0)=27 \longrightarrow 27,82,41,124,62,31,94,47,142,72 \rightsquigarrow Lukt\ Niet$

Kern 3
Somrij & Verschilrij

18)

Laag	*Aantal Stenen'*	*Driehoeksgetal*
1	1	1
2	2	1+2=3
3	3	1+2+3=6
4	4	6+4=10
5	5	10+5=15
6	6	15+6=21
7	7	21+7=28
8	8	28+8=36
9	9	36+9=45
10	10	45+10=55

19a) $t(n) = 2n+1$

n	0	1	2	3	4	5
$2n+1$	1	3	5	7	9	11
$S(n)$	1	4	9	16	25	36

19b) $t(n) = 6n-1$

n	0	1	2	3	4	5
$6n-1$	-1	5	11	17	23	29
$S(n)$	-1	4	15	32	55	84

19c) $t(n) = 2^n$

n	0	1	2	3	4	5
2^n	1	2	4	8	16	32
$S(n)$	1	3	7	15	31	63

19d) $t(n) = n^3$

n	0	1	2	3	4	5
n^3	0	1	8	27	64	125
$S(n)$	0	1	9	36	100	225

20a) $1{,}11{,}21{,}31{,}41{,}51 \longrightarrow S(5) = 156$
20b) $200{,}175{,}150{,}125{,}100{,}75 \longrightarrow S(5) = 825$
20c) $1{,}3{,}9{,}27{,}81{,}243 \longrightarrow S(5) = 364$
20d) $1,\ \frac{1}{2} = \frac{16}{32},\ \frac{1}{4} = \frac{8}{32},\ \frac{1}{8} = \frac{4}{32},\ \frac{1}{16} = \frac{2}{32},\ \frac{1}{32} \longrightarrow S(5) = 1\frac{31}{32}$
21a&b) $t(n)$ $\quad S(n) = 2n^2+5n+3$

n	0	1	2	3	4	5
$S(n)$	3	10	21	36	55	78
$t(n)$	3	7	11	15	19	23 $\xleftarrow{Telkens\ 4\ erbij}$

21c) $t(n) = 4n+3$
22a) $21^2 = 20^2+20+21 = 400+20+21 = 441$
22b) $31^2 = 30^2+30+31 \Rightarrow 31^2-30^2 = 30+31 = 61$
22c)

n	0	1	2	3	4	5	6	7	8	9	10
$t(n)$	0	1	4	9	16	25	36	49	64	81	100
$t(n)-t(n+1)$		1	3	5	7	9	11	13	15	17	19

23a) $t(n0 = 2n^2 - n$

n	0	1	2	3	4	5	6	7	8
$t(n)$	0	1	6	15	28	45	66	91	120
$v(n)$		1	5	9	13	17	21	25	29

23b) $t(n) = 5n+10$

n	0	1	2	3	4	5	6	7	8
$t(n)$	10	15	20	25	30	35	40	45	50
$v(n)$		5	5	5	5	5	5	5	5

23c) $t(n) = 2^n$

n	0	1	2	3	4	5	6	7	8
$t(n)$	1	2	4	8	16	32	64	128	256
$v(n)$		1	2	4	8	16	32	64	128

23d) $t(n) = \frac{1}{3}(n^3 - n)$

n	0	1	2	3	4	5	6	7	8
$t(n)$	0	0	2	8	20	40	70	112	168
$v(n)$		0	2	6	12	20	30	42	56

24a) $t(n) = n^2 \quad v(n) = 2n - 1$

n	0	1	2	3	4	5	6
$t(n)$	0	1	4	9	16	25	36
$v(n)$		$1 \xrightarrow{+2}$	$3 \xrightarrow{+2}$	$5 \xrightarrow{+2}$	$7 \xrightarrow{+2}$	$9 \xrightarrow{+2}$	11

Of:
$t(n) = n^2$
$v(n) = t(n) - t(n-1) \Rightarrow$
$v(n) = n^2 - (n-1)^2 \Rightarrow$
$v(n) = n^2 - (n-1)(n-1) \Rightarrow$
$v(n) = n^2 - (n^2 - 2n + 1)$
$v(n) = n^2 - n^2 + 2n - 1 \Rightarrow$
$v(n) = 2n - 1$

24b) $t(n) = n^2 + 10$
$v(n) = t(n) - t(n-1) \Rightarrow$
$v(n) = n^2 + 10 - (n-1)^2 - 10 \Rightarrow$
$v(n) = n^2 - (n-1)^2 \xrightarrow{\textit{zie}\ \text{Som 24a)}} v(n) = 2n - 1$

24c) $t(n) = n(n+2) \Rightarrow t(n) = n^2 + n$
$v(n) = t(n) - t(n-1) \Rightarrow$
$v(n) = n^2 + 2n - \left\{(n-1)^2 + 2(n-1)\right\} \Rightarrow$
$v(n) = n^2 + 2n - (n-1)^2 - 2(n-1) \Rightarrow$
$v(n) = n^2 + 2n - n^2 + 2n - 1 - 2n + 2 \Rightarrow$
$v(n) = 2n + 1$

24d) $t(n) = 100 - 3n$
$v(n) = t(n) - t(n-1) \Rightarrow$
$v(n) = 100 - 3n - \{100 - 3(n-1)\} \Rightarrow$
$v(n) = 100 - 3n - 100 + 3(n-1) \Rightarrow$
$v(n) = 100 - 100 - 3n + 3n - 3 \Rightarrow$
$v(n) = -3$

25) $t(n+2) = t(n+1) + t(n)$ en $t(0) = t(1) = 1$

n	1	2	3	4	5	6	7	8
$t(n+2)$	3	5	8	13	21	34	55	89
$v(n+2)$	1	2	3	5	18	13	21	34

KERN 4
REKENKUNDIGE RIJ

26)

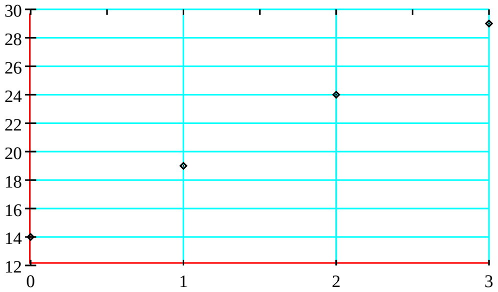
x	y	$Verschil$	y	$Verschil$	y	$Verschil$
0	**2**		**20**		**5**	
1	**4**	**2**	**16**	**-4**	**7,5**	**2,5**
2	**6**	**2**	**12**	**-4**	**10**	**2,5**
3	**8**	**2**	**8**	**-4**	**12,5**	**2,5**
4	**10**	**2**	**4**	**-4**	**15**	**2,5**
5	**12**	**2**	**0**	**-4**	**17,5**	**2,5**

27a) Rekenkundig; $v = 3$
27b) Niet rekenkundig
27c) Rekenkundig; $v = -5$
27d) Rekenkundig; $v = -2$

27e) Rekenkundig; $v = 4$ (2,6,10,14 etc)
27f) Rekenkundig; $v = \frac{1}{2}$
27g) Niet rekenkundig $\xrightarrow{Geen\ lineair\ Verband}$
27h) Rekenkundig; $v = 10$ (4,14,24,34 etc)

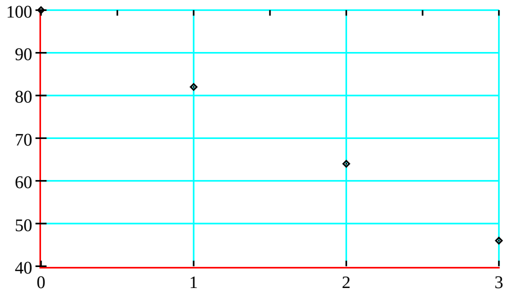
28a) $t(n) = 5n + 14$

plaatje bij Som 28a)



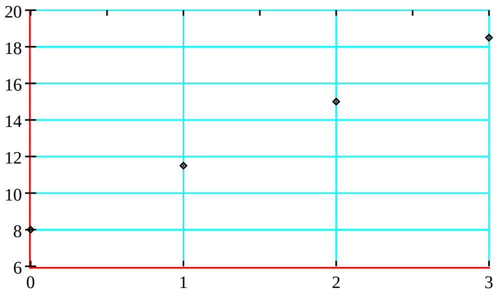
28c) $t(n) = -18n + 100$

plaatje bij Som 28c)



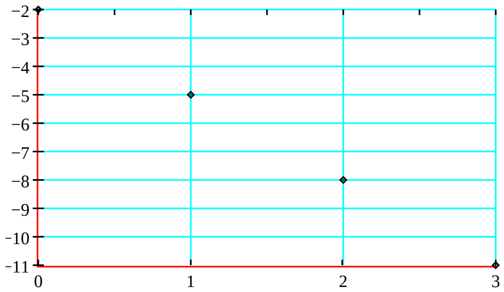
28b) $t(n) = 3\frac{1}{2}n + 8$

plaatje bij Som 28b)



28d) $t(n) = -3n - 2$

plaatje bij Som 28d)



29a&b)

$\dashrightarrow t(n) = 5n + 14 \longrightarrow t(50) = 264$

$\xrightarrow{Recursie\ Formule} t(n+1) = 5(n+1) + 14 \Rightarrow t(n+1) = 5n + 5 + 14 \Rightarrow t(n+1) = t(n) + 5 \ met\ t(0) = 14$

$\dashrightarrow t(n) = 3\frac{1}{2}n + 8 \longrightarrow t(50) = 3\frac{1}{2} \cdot 50 + 8 = 183$

$\xrightarrow{Recursie\ Formule} t(n+1) = t(n) + 3\frac{1}{2} \ met\ t(0) = 8$

$\dashrightarrow t(n) = -18n + 100 \longrightarrow t(50) = -18 \cdot 50 + 100 = -800$

$\xrightarrow{Recursie\ Formule} t(n+1) = t(n) - 18 \ met\ t(0) = 100$

$\dashrightarrow t(n) = -3n - 2 \longrightarrow t(50) = -3 \cdot 50 - 2 = -152$

$\xrightarrow{Recursie\ Formule} t(n+1) = t(n) - 3 \ met\ t(0) = -2$

30a) 1,2,3,4,5,6,7,8,9,10,11,12, 13,14,15,16,17,18,19,20
30b) 1+20=21
30c) 19+2=21

30d) 10 paren getallen met som 21

30e) 50 paren getalen met som 101

$101 \cdot 50 = 5050$

31a) $1000 + 1 = 1001$
$500 \cdot 1001 = 500500$

31b) $500 \cdot 2002 = 1001000$

32a)
$t(0) = 1$
$t(20) = 41$
$2 \cdot S(20) = 21 \cdot 42 \Rightarrow S(20) = \frac{21 \cdot 42}{2} = 441$

32b)
$t(0) = 200$
$t(20) = 100$
$2 \cdot S(20) = 21 \cdot 300 \Rightarrow S(20) = \frac{21 \cdot 300}{2} = 3150$

32c)
$t(n) = 8n + 4 \quad t(0) = 4$
$t(20) = 164$
$S(20) = \frac{21 \cdot 168}{2} = 1764$

32d)
$t(n) = -10n + 1000 \qquad t(0) = 1000$
$t(20) = 800$
$S(20) = \frac{21 \cdot 1800}{2} = 18900$

33)
$t(n) = 6n + 96 \qquad t(0) = 96$
$t(54) = 420$
$S(54) = \frac{55 \cdot (420+96)}{2} = 14190$

KERN 5
MEETKUNDIGE RIJ

34a)

n	0	1	2	3	4	5	6	7	8	9	10	11
$t(n)$	$1\xrightarrow{\times 2}$	$2\xrightarrow{\times 2}$	$4\xrightarrow{\times 2}$	$8\xrightarrow{\times 2}$	$16\xrightarrow{\times 2}$	$32\xrightarrow{\times 2}$	64	$128\xrightarrow{\times 2}$	256	512	$1024\xrightarrow{\times 2}$	2048

34b) $t(n) = 2^n$
35a) Meetkundig; $r = 3$
35b) Meetkundig; $r = \frac{1}{2}$
35c) Niet Meetkundig
35d) Meetkundig; $r = 2$
35e) Niet Meetkundig
35f) Niet Meetkundig

36a) $4 , 2 , 1 , \frac{1}{2} , \frac{1}{4} , \frac{1}{8}$
36b) $1000 , 100 , 10 , 1 , \frac{1}{10} , \frac{1}{100}$
36c) 1024 , 1280 , 1600 , 2000 , 2500 , 3125
36d) $a , ap , ap^2 , ap^3 , ap^4 , ap^5$

37a)

n	$t(n)$	$S(n)$	$T(n) = S(n)+1$
1	1	1	2
2	2	3	4
3	4	7	8
4	8	15	16
5	16	31	32
6	32	63	64
7	64	127	128
8	128	255	256

37b) Neen
37c) Ja
37d) $T(n) = 2^n$
37e)
$S(n) = T(n) - 1 = 2^n - 1$
$S(64) - 2^{64} - 1 \simeq 1,84 \times 10^{19}$

38a) 32
38b)

Ronde	1	2	3	4	5	6
Wedstrijden	32	16	8	4	2	1

38c) $t(n) = 64 \cdot \left(\frac{1}{2}\right)^n$
38d) 32+16+8+4+2+1=63
38e) 63
38f) 127

39a) $\frac{1}{2} + \frac{1}{4} = \frac{3}{4}$

39b)

Ronde	1	2	3	4	5
Per Ronde Zwart	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$	$\frac{1}{32}$
Totaal Zwart	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{7}{8}$	$\frac{15}{16}$	$\frac{31}{32}$

40a) $2^1 = 2,\ 2^2 = 4;\ 2^3 = 8;\ 2^4 = 16 \dashrightarrow 2^{10} = 1024$
na 10 rondes: $\xrightarrow{Wit} \frac{1}{1024}$
na 10 rondes: $\xrightarrow{Zwart} \frac{1023}{1024}$
40b) Het witte gedeelte halveert steeds $\rightsquigarrow \left(\frac{1}{2}\right)^n$
Het totale oppervlakte is : 1
Voor het zwart gedeelte geldt dus : $1 - \left(\frac{1}{2}\right)^n$

41a) $t(n) = 3^n$
$S(20) = \frac{t(21)-t(0)}{34} = \frac{3^{21}-3^0}{2} = 5230176601$
41b) $t(n) = 0,5^n$
$S(20) = \frac{t(21)-t(0)}{0,5-1} = \frac{0,5^{21}-0,5^0}{-0,5} \simeq 2,00$
41c) $t(n) = 1000 \cdot 1,04^n$
$S(20) = \frac{t(21)-t(0)}{1,04-1} = \frac{1000 \cdot 1,04^{21}-1000}{0,04} \simeq 31969$
41d) $t(n+1) = 10 \cdot t(n);$
$t(0) = 1 \Rightarrow t(n) = 10^n$
$S(20) = \frac{t(21)-t(0)}{10-1} = \frac{10^{21}-1}{9} \simeq 1,1 \times 10^{20}$
41e) $t(n+1) = 0,1 \cdot t(n),$
$t(0) = 1 \Rightarrow t(n) = 0,1^n$
$S(20) = \frac{t(21)-t(0)}{0,1-1} = \frac{0,1^{21}-1}{-0,9} \simeq 1,11$
41f) $t(n+1) = 1,1 \cdot t(n)$,
$t(0) = 20 \Rightarrow t(n) = 20 \cdot 1,1^n$
$S(20) = \frac{t(21)-t(0)}{1,1-1} = \frac{20 \cdot 1,1^{21}-20}{0,1} \simeq 1280$

42a)

belronde	1	2	3	4	5	6	7
wordt gebeld	1	3	9	27	81	243	729
Totaal	1	4	13	40	121	364	1093

42b) 121

42c)

4 belrondes bij 30 deelnemers

5 belrondes bij 60 deelnemers

7 belrondes bij 600 deelnemers

43a) elk jaar komt er 4% rente bij waarover hij het volgende jaar ook weer rente trekt

$4\% erbij \Rightarrow r = 1,04 \longrightarrow t(n) = 1000 \cdot 1,04^n$

43b)

1^{ste}: $1000 \cdot 1,04^{10}$

2^{de}: $1000 \cdot 1,04^{9}$

3^{de}: $1000 \cdot 1,04^{8}$

4^{de}: $1000 \cdot 1,04^{7}$

5^{de}: $1000 \cdot 1,04^{6}$

$\downdownarrows \quad \downdownarrows$

9^{de} : $1000 \cdot 1,04^{2}$

43c) Totaal: Hfl. 11446,35

$S(10) = \frac{t(11)-t(0)}{1,04-1} = \frac{1000 \cdot 1,04^{11} - 1000}{0,04} \simeq 13486,35 \longrightarrow$

$13486,35 - 1000 - 1000 \cdot 1,04 \simeq 11446,35$

1^{ste}	1000,-	$\xrightarrow{Na\ 10\ jaar}$	$1000 \cdot 1,04^{10}$	$t(8)$
2^{de}	1000,-	$\xrightarrow{Na\ 9\ jaar}$	$1000 \cdot 1,04^{9}$	$t(7)$
3^{de}	1000,-	$\xrightarrow{Na\ 8\ jaar}$	$1000 \cdot 1,04^{8}$	$t(6)$
4^{de}	1000,-	$\xrightarrow{Na\ 7\ jaar}$	$1000 \cdot 1,04^{7}$	$t(5)$
5^{de}	1000,-	$\xrightarrow{Na\ 6\ jaar}$	$1000 \cdot 1,04^{6}$	$t(4)$
6^{de}	1000,-	$\xrightarrow{Na\ 5\ jaar}$	$1000 \cdot 1,04^{5}$	$t(3)$
7^{de}	1000,-	$\xrightarrow{Na\ 4\ jaar}$	$1000 \cdot 1,04^{4}$	$t(2)$
8^{de}	1000,-	$\xrightarrow{Na\ 3\ jaar}$	$1000 \cdot 1,04^{3}$	$t(1)$
9^{de}	1000,-	$\xrightarrow{Na\ 2\ jaar}$	$1000 \cdot 1,04^{2}$	$t(0)$
		Totaal	11446,35	

Of met Formule:

$t(n) = 1000 \cdot 1,04^2 \cdot 1,04^n$

$S(n) = \frac{t(n+1)-t(0)}{r-1} \longrightarrow$

$S(8) = \frac{1000 \cdot 1,04^2 \cdot 1,04^9 - 1000 \cdot 1,04^2}{0,04} = 11446,35$

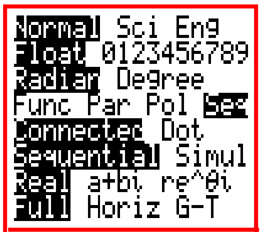
Grafische Rekenmachine

G1a&b)

2 , 7 , 12 , 17 , 21 , 27 , 32

G1c)

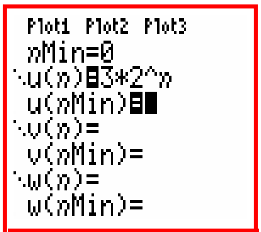
$t(n) = 2 + 5n$



Screendump Ti83

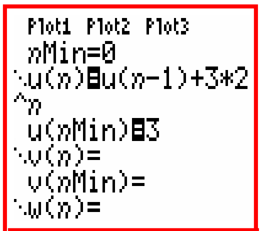
Som G1

G2) $t(n) = 3 \cdot 2^n$



Screendump Ti83

Som G2



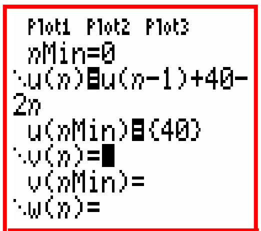
Screendump Ti83

Som G2c

⇉Invoeren $\longrightarrow y =$
$\longrightarrow u(n) = 3 \cdot 2^n$
$\longrightarrow nMin = 0$
⇉**Invoeren:** $\longrightarrow nMin = 0$
$u(n) = u(n-1) + 3 \cdot 2$
$u(nMin) = 3$
$n = 9 \xleftarrow{kijken bij \boxed{table}}$

G2a) 3 , 6 , 12 , 24 , 48 , 96
G2b)
$t(9) = 1536$
$t(15) = 98304$
G2c) 3069

G3a) $t(n) = 40 - 2n$

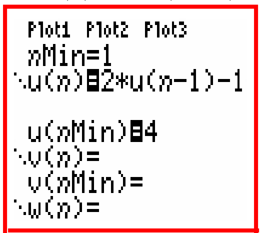


Screendump Ti83

Som G3a

⇉**Invoeren:** $\longrightarrow nMin = 0$
$u(n) = u(n-1) + 40 - 2n$
$u(nMin) = 40$
$\xrightarrow{Aflezen bij} n = 11$
G3b) 348 tegels
$\xrightarrow{Kontrole} S(11) = 12 \cdot \left(\frac{40+18}{2}\right) = 348$

G4) $t(n) = 2 \cdot t(n-1) - 1 \quad t(1) = a$



Screendump Ti83

Som G4

G4a)
$a = 4 \longrightarrow t(1) = 4$
$\rightrightarrows$**Invoeren:** $y = \longrightarrow nMin = 1 \longrightarrow$
$u(n) = 2 \cdot u(n-1) - 1$
$u(nMin) = 4$

$\boxed{Quit} \longrightarrow Gewone\ Scherm \xrightarrow{Intikken} \begin{cases} u(3) = \\ u(30) = \end{cases}$
$t(3) = 13$
$t(30) = 1610612737 \simeq 1,61 \times 10^9$
G4b)
$\rightrightarrows invoeren \longrightarrow \begin{cases} u(nMin) = 4 \\ u(nMin) = 1 \end{cases}$
$t(3) = 1$
$t(30) = 1$
G4c)
$t(7) = 2 \cdot t(6) - 1 = 385 \Rightarrow t(6) = 193$
$t(6) = 2 \cdot t(5) - 1 = 193 \Rightarrow t(5) = 97$
$t(5) = 2 \cdot t(4) - 1 = 97 \Rightarrow t(4) = 49$
$t(4) = 2 \cdot t(3) - 1 = 49 \Rightarrow t(3) = 25$
$t(3) = 2 \cdot t(2) - 1 = 25 \Rightarrow t(2) = 13$
$t(2) = 2 \cdot t(1) - 1 = 13 \Rightarrow t(1) = 7 \xrightarrow{Dus} a = 7$

G5a) Afname 20% $\xrightarrow{BlijvenOver}$ 80% $\longrightarrow g = 0,8$
$u(0) = 5000$
$u(1) = 5000 \cdot 0,8 + 1100$
$u(2) = (5000 \cdot 0,8 + 1100) \cdot 0,8 + 1100$
$\downdownarrows \quad \downdownarrows$
$u(n) = u(n-1) \cdot 0.8 + 1100$
G5b) $u(6) \simeq 5369$ kippen
G5c) Hij begon met 5000 kippen. Er kwamen 6×1100 kippen bij.
Hij heeft dus $5000 + 6600 = 11600$ kippen aangeschaft.
Hij heeft zijn afnemers $11600 - 5369 = 6231$ kippen geleverd.
G6a) Aflossingsdeel $\frac{90000}{360} = Hfl\,250,-$
G6b) 1^{ste}Aflossing= vaste deel + rente deel
Dus:
$B(1) = 250 + 90000 \cdot 0,007 = Hfl\,880,-$
$B(2) = 250 + (90000 - 250) \cdot 0,007$
$B(3) = 250 + (90000 - 2 \cdot 250) \cdot 0,007$
$B(4) = 250 + (90000 - 3 \cdot 250) \cdot 0,007$
$\downdownarrows \quad \downdownarrows \quad \downdownarrows$
$B(n) = 250 + (90000 - (n-1) \cdot 250) \cdot 0,007 \Rightarrow$
$B(n) = 250 + 90000 \cdot 0,007 - (n-1) \cdot 250 \cdot 0,007 \Rightarrow$
$B(n) = 880 - n \cdot 250 \cdot 0,007 + 1 \cdot 250 \cdot 0,007 \Rightarrow$
$B(n) = 881,75 - 1,75n$
G6c)
$n = 1 \longrightarrow 1\ feb.1990$
$n = 13 \longrightarrow 1\ feb.1991 \xleftarrow{1 \times 12\ Maand\ Later}$
$n = 121 \longrightarrow 1\ feb.2000 \xleftarrow{10 \times 12\ Maand\ Later}$
$B(121) = 881,75 - 1,75 \cdot 121 = 670$
G6d) $B(n) = 881,75 - 1,75n$
Per maand is het rentedeel gelijk aan:
$881,75 - 1,75 \cdot n - 250 = 631,75 - 1,75 \cdot n$
Hiervan krijg je 50% terug, dus netto betaal je de helft, en dat is
$\frac{1}{2}(631,75 - 1,75 \cdot n)$
Netto Maandlast=$250 + \frac{1}{2}(731,75 - 1,75n) = 565,875 - 0,875n$
G6e) $90.000 \cdot 0,007 = Hfl\,630,-$

G6f)

$A(1) = 800 - 90.000 \cdot 0,007 = Hfl\,170,-$

$A(2) = 800 - (90.000 - A(1)) \cdot 0,007 = 800 - S(1) \cdot 0,007$

$A(3) = 800 - S(2) \cdot 0,007$

$A(4) = 800 - S(3) \cdot 0,007$

$\downdownarrows \qquad \downdownarrows$

$A(n) = 800 - S(n-1) \cdot 0,007$

Het Rentedeel is 7% van de aanwezige schuld

$\longrightarrow$ ***Schuld=oudeschuld-aflossingsdeel*** $\longrightarrow S(n) = S(n-1) - A(n) \Rightarrow$

$\Rightarrow S(n) = S(n-1) - 800 + 0,007 \cdot S(n-1) = 1,007 \cdot S(n-1) - 800$

G6g)

$S(0) = 90.000$

$S(12) = 1,007 \cdot S(11) - 800 = Hfl\,87.879,60$

$S(222) \simeq 26$

$S(223) \simeq -774$

Looptijd is 223 maanden

G6i) Stel maandbedrag is M

$\Rightarrow A(n) = M - S(n-1) \cdot 0,007 \Rightarrow S(n) = 1,007 \cdot S(n-1) - M$

$S(0) = 90.000$

$S(360) \simeq 0$

$S(360) = 1,007 \cdot S(359) - M = 0 \Rightarrow$

$$\left.\begin{array}{c} M = 1,007 \cdot S(359) \\ S(359) = 1.007 \cdot S(358) - M \end{array}\right\} \Rightarrow \left.\begin{array}{c} M = 1,007 \cdot (1,007 \cdot S(358) - M) \Rightarrow \\ \Rightarrow M = 1,007^2 \cdot S(358) - 1,007 \cdot M \end{array}\right\} \Rightarrow \left.\begin{array}{c} 2,007 \cdot M = 1,007^2 \cdot S(358) \\ S(358) = 1,007 \cdot S(357) - M \end{array}\right\} \Rightarrow$$

$\Rightarrow 2,007 \cdot M = 1,007^2 \cdot (1,007 \cdot S(357) - M) = 1,007^3 \cdot S(357) - 1,007^2 M \Rightarrow$

$(1 + 1,007 + 1,007^2)M = 1,007^3 \cdot S(357)$

$\downdownarrows \qquad \downdownarrows$

$(1 + 1,007 + 1,007^2 + 1,007^3 + 1,007^4 + 1,007^5 \star\star\star\star\star\star 1,007^{359}) \cdot M = 1,007^{360} \cdot S(0) = 1108797$

$som = \frac{1,007^{360} - 1}{1,007 - 1} \simeq 1617,137$

$M = \frac{1,007^{360} \cdot 90.000}{1617,137} \simeq Hfl\,686,-$

$1^{ste} maand$	90.000×0,007=630	$\frac{1}{2} \cdot 90.000 \cdot 0,007 = Hfl\,315,-$
1^{ste}	$800 - \frac{1}{2} \cdot S(0) \cdot 0,007$	
2^{de}	$800 - \frac{1}{2} \cdot S(1) \cdot 0,007$	$= 800 - \frac{1}{2} \cdot (1,007 \cdot S(0) - 800) \cdot 0,007$
3^{de}	$800 - \frac{1}{2} \cdot S(2) \cdot 0,007$	$= 800 - \frac{1}{2} \cdot (1,007 \cdot S(1) - 800) \cdot 0,007$
4^{de}	$800 - \frac{1}{2} \cdot S(3) \cdot 0,007$	$= 800 - \frac{1}{2} \cdot (1,007 \cdot S(2) - 800) \cdot 0,007$
5^{de}	$800 - \frac{1}{2} \cdot S(4) \cdot 0,007$	$= 800 - \frac{1}{2} \cdot (1,007 \cdot S(3) - 800) \cdot 0,007$
$\downdownarrows$	$\downdownarrows$	$\downdownarrows$
$n^{de}\ n \geqq 2$	$800 - \frac{1}{2} \cdot S(n-1) \cdot 0,007$	$= 800 - \frac{1}{2} \cdot (1,007 \cdot S(n-2) - 800) \cdot 0,007 = 800 - \frac{1}{2} \cdot (1,007 \cdot S(0) - 800) \cdot 0,007$

DOORWERKING

D1a)

Omtrek $P_0 = 3\cdot 6 = 18$

Omtrek $P_1 = 3\cdot 8 = 24$

D1b)

0^{de}	Model	3	=3	Lijnstukken
1^{ste}	Model	$3\cdot 4$	=12	Lijnstukken
2^{de}	Model	$3\cdot 4\cdot 4$	=48	Lijnstukken
3^{de}	Model	$3\cdot 4\cdot 4\cdot 4$	=192	Lijnstukken
4^{de}	Model	$3\cdot 4\cdot 4\cdot 4\cdot 4$	=768	Lijnstukken
5^{de}	Model	$3\cdot 4^5$	=3072	Lijnstukken

D1c)

$$Omtrek\ P_0 \quad 3\cdot 6 \quad = 3\cdot 6\cdot \frac{4^0}{3^0}$$
$$Omtrek\ P_1 \quad 3\cdot 4\cdot \frac{6}{3} \quad = 3\cdot 6\cdot \frac{4^1}{3^1}$$
$$Omtrek\ P_2 \quad 3\cdot 4^2\cdot \frac{6}{3\cdot 3} \quad = 3\cdot 6\cdot \frac{4^2}{3^2}$$
$$Omtrek\ P_3 \quad 3\cdot 4^3\cdot \frac{6}{3\cdot 3\cdot 3} \quad = 3\cdot 6\cdot \frac{4^3}{3^3}$$
$$Omtrek\ P_4 \quad 3\cdot 4^4\cdot \frac{6}{3\cdot 3\cdot 3\cdot 3} \quad = 3\cdot 6\cdot \frac{4^4}{3^4}$$
$$Omtrek\ P_n \quad 3\cdot 4^n\cdot \frac{6}{3^n} \quad = 3\cdot 6\cdot \frac{4^n}{3^6} = 18\left(\frac{4}{3}\right)^n$$

D1d)

$P_{29} = 18\cdot\left(\frac{4}{3}\right)^{29} \simeq 75595$

$P_{30} = 18\cdot\left(\frac{4}{3}\right)^{30} \simeq 100794$

kleinste waarde $n = 30$

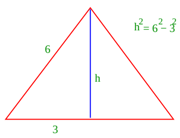
D1e)

$n = 0 \Rightarrow O_0 = \left(1,6-0,6\cdot\left(\frac{4}{9}\right)^0\right)\cdot 9\sqrt{3} = 9\sqrt{3}$

$h^2 = 6^2-3^2 = 36-9 = 27 = 9\cdot 3$

$h = \sqrt{9\cdot 3} = \sqrt{9}\cdot\sqrt{3} = 3\sqrt{3} \Rightarrow$

$Opp_{P_0} = \frac{1}{2}\cdot 6\cdot 3\cdot\sqrt{3} = 9\sqrt{3} \xleftarrow{Klopt}$



$n = 1 \Rightarrow O_1 = \left(1,6-0,6\cdot\left(\frac{4}{9}\right)^1\right)\cdot 9\sqrt{3}$

$O_1 = \left(1,6-\frac{6}{10}\cdot\frac{4}{9}\right)\cdot 9\sqrt{3}$

$O_1 = \left(1\frac{6}{10}-\frac{24}{90}\right)\cdot 9\sqrt{3}$

$O_1 = \left(1\frac{54}{90}-\frac{24}{90}\right)\cdot 9\sqrt{3} = 1\frac{30}{90}\cdot 9\sqrt{3}$

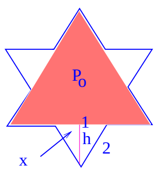
$O_1 = 1\frac{1}{3}\cdot 9\sqrt{3} = 12\sqrt{3}$

$Opp_{P_1} = Opp_{P_0} + 3Opp_x$

$h^2 = 2^2-1^2 = 3 \Rightarrow h = \sqrt{3} \longrightarrow$

$Opp_x = \frac{1}{2}\cdot 2\cdot\sqrt{3} = \sqrt{3}$

$\xrightarrow{Dus} Opp_{P_1} = 9\sqrt{3}+3\sqrt{3} = 12\sqrt{3} \xleftarrow{Klopt}$



D1f)

$\left(1,6-0,6\cdot\left(\frac{4}{9}\right)^n\right)\cdot 9\sqrt{3} = 25 \Rightarrow 1,6-0,6\left(\frac{4}{9}\right)^n = \frac{25}{9\sqrt{3}} \Rightarrow 0,6\cdot\left(\frac{4}{9}\right)^n = 1,6-\frac{25}{9\sqrt{3}} \Rightarrow \left(\frac{4}{9}\right)^n = -0,006 \xleftarrow{Kan\ Niet}$

$\left(1,6-0,6\cdot\left(\frac{4}{9}\right)^n\right)\cdot 9\sqrt{3} = 24 \Rightarrow 1,6-0,6\cdot\left(\frac{4}{9}\right)^n = \frac{24}{9\sqrt{3}} \Rightarrow 0,6\cdot\left(\frac{4}{9}\right)^n = 1,6-\frac{24}{9\sqrt{3}} \Rightarrow \left(\frac{4}{9}\right)^n \simeq 0,1007 \xleftarrow{Kan\ Wel}$

D2a) $365,2422\cdot 900 \simeq 328717,98$ dagen

$\frac{900}{4} = 225 \Rightarrow \left.\begin{array}{l} 225 \; jaar \; met \; 366 \, dagen \\ 675 \; jaar \; met \; 365 \, dagen \end{array}\right\} \Rightarrow 225 \cdot 366 + 675 \cdot 365 = 328725 \xrightarrow{Verschil} 7,02 \, dagen$

D2b) 3 schikkeljaren worden omgezet dus 4 eeuwen bestaan dan uit
$400 \cdot 365 + 100 - 3 = 146097 \; dagen$
Werkelijk; $400 \cdot 365,2422 \simeq 146096,88 \, dagen \xrightarrow{Verschil} 0,12 \; dagen$

D2c)
$r(23:7) = 2$
$r(75:13) = 10$
$r(5:8) = 5$

D2d)
$x = 2005$
$y_1 = 19 \cdot r(2005:19) + 24 \Rightarrow y_1 = 19 \cdot 10 + 24 = 214$
$y_2 = r(214:30) = 4$
$y_3 = 6 \cdot 4 = 24$
$y_4 = 4 \cdot r(2005:7) = 4 \cdot 3 = 12$
$y_5 = 2 \cdot r(2005:4) = 2 \cdot 1 = 2$
$y_6 = r((24+12+2+5):7) = r(43:7) = 1$
$n = 4+1+1 = 6$
$21 + 6 = 27$ maart