

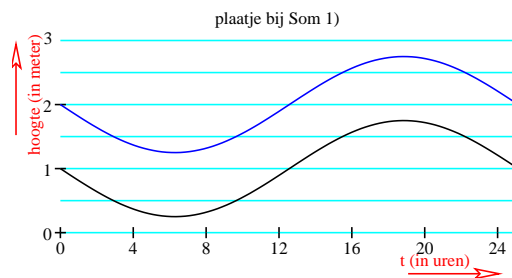
# UITWERKINGEN VOOR HET VWO A1B1 DEEL1

## Hoofdstuk 7

### WERKEN MET GRAFIEKEN

#### KERN 1 VERTIKAAL

1a)



1b) 1 omhoog schuiven

2a)  $g(x) = \cos(x) + 1\frac{1}{2}$

2b)  $h(x) = \cos(x) - 2$

2c) De grafiek van  $g(x)$   $3\frac{1}{3}$  omlaag schuiven

3a)  $f(x) = x^2 - 4x$  *Snijpunt met x-as*

$$0 = x^2 - 4x \Rightarrow x(x-4) = 0 \Rightarrow x(x-4) = 0 \Rightarrow x = 0 \vee x = 4$$

$$\Rightarrow x = 0 \vee x = 4$$

*Symmetrie-As*  $\rightarrow x = 2 \Rightarrow f(2) = 2^2 - 4 \cdot 2 = -4$

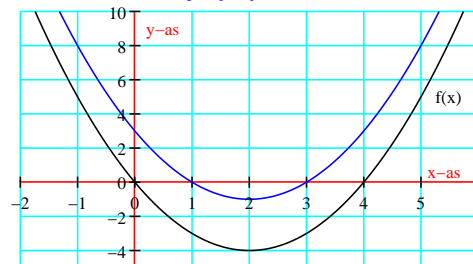
$x$	1	5	6
$f(x)$	-3	5	12

3b)

$$\left. \begin{array}{l} (1; f(1)) = (1; -3) \\ (3; f(3)) = (3; -3) \end{array} \right\} \xrightarrow{3 \text{ omhoog dan...}} \left\{ \begin{array}{l} (1; 0) \\ (3; 0) \end{array} \right.$$

3c) 4 omhoog

plaatje bij Som 3)



<sup>1</sup> Deze samenvatting mag niet massaal op kosten van Schaersvoorde worden Uitgeprint!!!



2

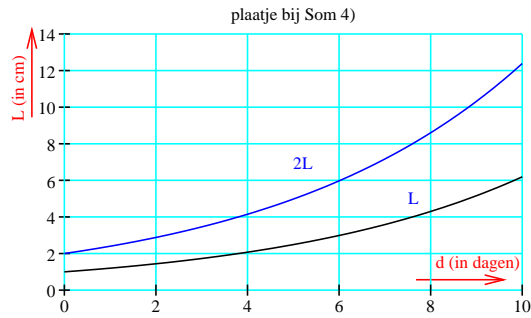
werd gemaakt onder Linux met  $\LaTeX$  en  $\text{\LaTeX}$

<sup>3</sup> Typ&andere fouten&blunders graag Melden!

4)  $L = 1, 2^d$

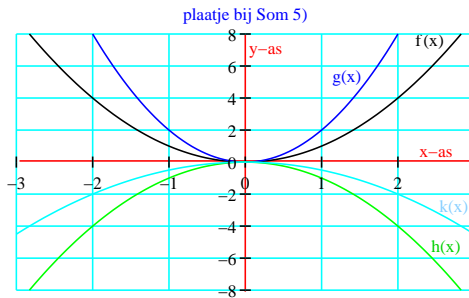
4a)

$d$	0	1	2	3	4	5	10
$L$	1	1,2	1,44	1,7	2,1	2,5	6,2
$2L$	2	2,4	2,9	3,4	4,2	5,0	12,4



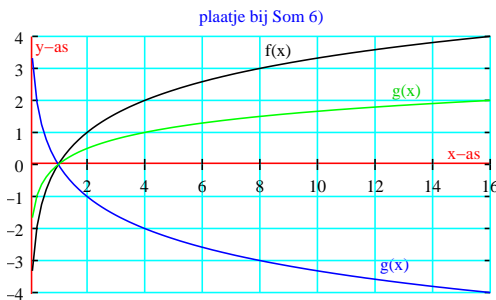
5a)  $f(x) = x^2 \ x \in [-3; 0]$

$d$	-3	-2	-1	0	1	2	3
$f(x)$	9	4	1	0	1	4	9
$g(x)$	18	8	2	0	2	8	18
$h(x)$	-9	-4	-1	0	-1	-4	-9
$k(x)$	-4,5	-2	-0,5	0	-0,5	-2	-4,5



6)

$x$	0,5	1	2	3	4	8	16
$f(x)$	-1	0	1	1,6	2	3	4
$g(x)$	1	0	-1	-1,6	-2	-3	-4
$h(x)$	-0,5	0	0,5	0,8	1	1,5	2



7)

Algemeen  $\rightarrow h(t) = a - b \sin(t) \left\{ \begin{array}{l} \text{periode: } 2\pi \quad t = t \\ \text{evenwichtslijn: } y = 4 \quad a = 4 \\ \text{amplitude: } 3 \quad b = 3 \end{array} \right\} \Rightarrow h(t) = 4 - 3 \cdot \sin(t)$

$t$	0	$\frac{1}{6}\pi \simeq 0,5$	$\frac{1}{2}\pi \simeq 1,6$	$\pi \simeq 3,14$	$1\frac{1}{2}\pi \simeq 4,7$	$2\pi \simeq 6,3$	
$\sin(t)$	0	0,5	1	0	-1	0	hier eerst omhoog, dan omlaag
$-\sin(t)$	0	-0,5	-1	0	1	0	hier eerst omlaag dan omhoog
$-3 \sin(t)$	0	-1,5	-3	0	3	0	Amplitude=3
$4 - 3 \sin(t)$	4	2,5	1	4	7	4	evenwichtslijn $y = 4$

8a) 110 cm of 130 cm

8b) lager dan 50

8c)  $Hf1\ 2370, 75$  of  $Hf1\ 2309, 25$

9a)  $|3| + |-2| = 5$

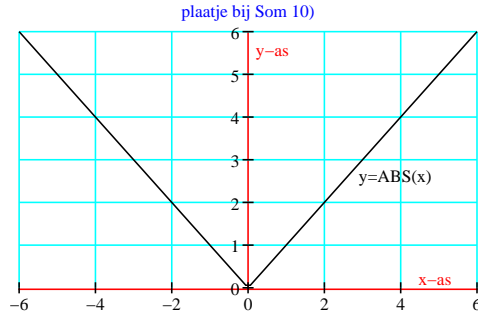
9b)  $ABS(4 - 18) = ABS(-14) = 14$

9c)  $|-4| + |7| = 11$

9d)  $|-4 + 7| = |-3| = 3$

10a)  $|-3| = 3$   $|0| = 0$   $ABS(4) = 4$

10b)  $ABS(x) = |x|$   $y = |x| = \begin{cases} x \text{ voor } x \geq 0 \\ -x \text{ voor } x \leq 0 \end{cases}$



11a)

Van  $(0;0)$  naar  $(x;y)$  Moet je  $\left\{ \begin{array}{l} x \text{ stappen naar rechts en } y \text{ stappen omhoog} \\ -x \text{ stappen naar links en } y \text{ stappen omhoog} \\ x \text{ stappen naar rechts en } -y \text{ stappen omlaag} \\ -x \text{ stappen naar links en } y \text{ stappen omlaag} \end{array} \right\} \Rightarrow |x| + |y| \text{ stappen} \Rightarrow$

$\Rightarrow \text{Afstand} = |x| + |y|$

12a)

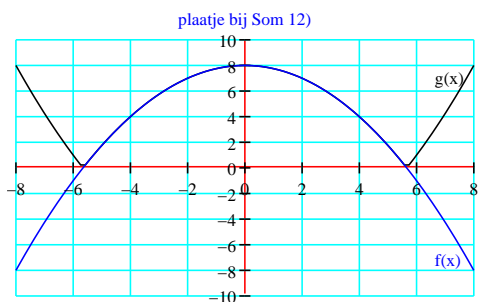
$8 - 0,25x^2 = 0 \Rightarrow 0,25x^2 = 8 \Rightarrow x^2 = 32 \Rightarrow x = \sqrt{32} \vee x = -\sqrt{32} \Rightarrow$   
 $\Rightarrow x = \sqrt{16} \cdot \sqrt{2} \vee x = -\sqrt{16} \cdot \sqrt{2} \Rightarrow \cdot \sqrt{2} \Rightarrow x = 4\sqrt{2} \vee x = -4\sqrt{2}$

$x$	-8	-4	0	4	8
$8 - 0,25x^2$	-8	4	8	4	-8
$ 8 - 0,25x^2 $	8	4	8	4	8

12c) Het gedeelte onder de  $x - as$  wordt gespiegeld in de  $x - as$

Of voor  $x \geq 4\sqrt{4}$  en  $x \leq -4\sqrt{4}$

$f(x) = 8 - 0,25x^2$  wordt  $-8 + 0,25x^2$



## KERN 2

### HORIZONTALAAL

**13a)** 22 min.

**13b)**  $g(t) = f(t - 22)$

Stel R. vertrekt altijd op  $t = 0 \xrightarrow{\text{Dan}} f(0) = 0 \xleftarrow{\text{0 meter afgelegd}}$

22 minuten later (Na verslapen) 0 meter afgelegd  $\rightarrow g(22) = f(22 - 22) = f(0) = 0$

**14)**  $f(x) = x^2$

**14a)** 1 naar Rechts

**14b)**  $g(x) = (x+)^2 - 1$   $f(x) = x^2$

2 naar Links, 3 naar Beneden

**14c)**  $g(x) = x^2 - 4x = (x - 2)^2 - 4 \xrightarrow{\text{Ofwel}} g(x) = x^2 - 4x = (x^2 - 4x + 4) - 4$

2 naar Rechts, 4 naar Beneden

**15)**

**I:**  $\left\{ \begin{array}{l} f(x) = x^2 \\ 2 \text{ naar links verschoven } I(x) = (x+2)^2 \\ (0;0) \text{ top van } f \quad f(0) = 0 = 0^2 \\ (-2;0) \text{ top van } I \quad I(-2) = (-2+2) = 0^2 = 0 \end{array} \right.$

$x$		-2		-1		0		1		2
$f(x) = x^2$		4		1		0		1		4
$I(x) = (x+2)^2$		0		1		4		1		4

**II:**  $\left\{ \begin{array}{l} h(x) = {}^2 \log x \\ 1 \text{ naar Rechts verschoven } I(x) = (x+2)^2 \\ h(1) = {}^2 \log 1 = 0 \\ II(2) = {}^2 \log(2-1) = {}^2 \log 1 = 0 \\ II(x) = {}^2 \log(x-1) \end{array} \right.$

$x$		$\frac{1}{2}$		1		2		4		5
$h(x) = {}^2 \log(x)$		-1		0		1		2		${}^2 \log 5$
$II(x)$		-		-		0		${}^2 \log 3$		${}^2 \log 4 = 2$

**III:**  $\left\{ \begin{array}{l} g(x) = \sqrt{x} \rightarrow x \geq 0 \\ 4 \text{ naar links } 4 \text{ omlaag} \\ x \geq -4 \quad III(x) = \sqrt{x+4} = -4 \end{array} \right.$

$x$		-4		-3		0		5
$g(x) = \sqrt{x}$		-		-		0		$\sqrt{5}$
$III(x)$		-4		-3		-2		-1

**16a)**  $f(x) = \sin(x)$

$x$		0		$\frac{1}{6}\pi$		$\frac{1}{2}\pi$		$\frac{5}{6}\pi$		$\pi$		$1\frac{1}{6}\pi$
$f(x) = \sin(x)$		0		$\frac{1}{2}$		1		$\frac{1}{2}$		0		$-\frac{1}{2}$

**16b)**  $g(x) = \sin(x - \frac{1}{3}\pi)$

$g(\frac{1}{3}\pi) = \sin(\frac{1}{3}\pi - \frac{1}{3}\pi) = \sin(0) = 0$

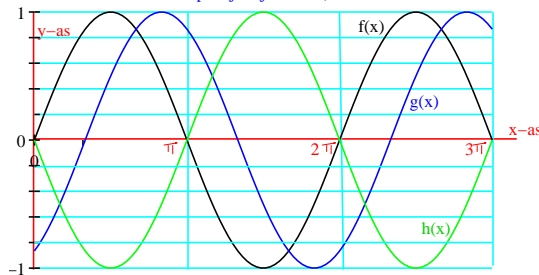
Dus verschuiven  $\frac{1}{3}\pi$  naar Rechts

**16c)**  $h(x) = \sin(x + \pi)$

$h(-\pi) = \sin(-\pi + \pi) = \sin(0) = 0$

Dus verschuiven  $\pi$  links

plaatje bij Som 16)



**17)**

$f : \left. \begin{array}{l} \text{periode is } 2\pi \\ \text{Amplitude is } 2 \end{array} \right\} \Rightarrow f(x) = 2 \sin(x)$

$f(x) \xrightarrow{\text{Hoogste } y\text{-waarde is } 2} 2$   
 $g(x) \xrightarrow{\text{hoogste } y\text{-waarde is } 5} 5$  }  $\rightarrow 3 \text{ omhoog en } \frac{1}{3}\pi \text{ naar rechts} \rightarrow g(x) = 2 \sin(x - \frac{1}{3}\pi) + 3$

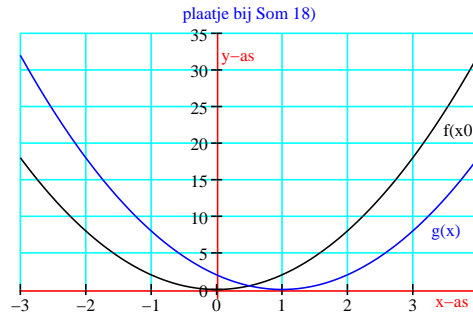
18a)  $f(x) = 2x^2$

$x$	-3	-2	-1	0	1	2	3
$f(x)$	18	8	2	0	2	8	18
$g(x)$		18	8	2	0	2	8

$f(0) = 2 \cdot 0^2 = 0 \rightarrow (0; 0)$

$g(1) = 2 \cdot (1 - 1)^2 = 0 \rightarrow (0; 0)$

$g(x) = 2(x - 1)^2$



19)

$$A: \left\{ \begin{array}{l} \text{periode} = \pi \xrightarrow{\text{Mogelijk zijn}} 1, 2, 3, 4, 5, 6 \\ \text{evenwichtsstand} : y = 0 \xrightarrow{\text{vallen af}} 1, 4 \\ \text{1}^{\text{ste}} \text{ snijpunt met de } x\text{-as} : (\frac{1}{2}; 0) \rightarrow \\ f(\frac{1}{2}) = \sin(2 \cdot \frac{1}{2} - 1) = \sin(0) = 0 \xrightarrow{\text{Dus}} 2 \end{array} \right\} \xrightarrow{\text{A hoort bij 2}} f(x) = \sin(2x - 1)$$

$$B: \left\{ \begin{array}{l} \text{periode} = \pi \xrightarrow{\text{Mogelijk zijn}} 1, 2, 3, 4, 5, 6 \\ \text{evenwichtsstand} : y = 0 \xrightarrow{\text{vallen af}} 1, 4 \\ (0; 0, 9) \xleftarrow{\text{is een Punt van B}} \end{array} \right\} \xrightarrow{\text{⑥}} f(x) = \sin 2(x + 1)$$

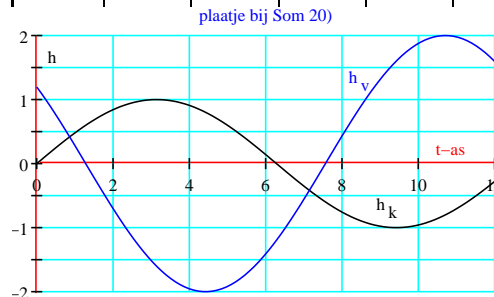
$$\left\{ \begin{array}{l} \text{②} \quad f(0) = \sin(2 \cdot 0 - 1) = \sin(-1) \simeq -0,8 \neq 0,9 \xleftarrow{\text{VALT AF}} \\ \text{③} \quad f(0) = \sin 2 \cdot (0 - 1) = \sin(-2) \simeq -0,9 \neq 0,9 \xleftarrow{\text{VALT AF}} \\ \text{⑤} \quad f(0) = \sin(2 \cdot 0 + 1) = \sin(1) \simeq 0,8 \neq 0,9 \xleftarrow{\text{VALT AF}} \\ \text{⑥} \quad f(0) = \sin 2(0 + 1) = \sin(2) \simeq 0,9 \end{array} \right.$$

20)  $h_k = \sin \frac{1}{2}k$  waarin  $k$  in meter,  $t$  in uren

20a)  $h_v = 2 \sin \frac{1}{2}(t + 5)$

20b)

$t$	1	2	3	4	5	6	7	8	9	12
$h_k$	0,48	0,84	0,997	0,91	0,6	0,14	-0,35	-0,76	-0,98	-0,28
$h_v$	0,28	-0,70	-1,51	-1,96	-1,92	-1,41	-0,56	0,43	1,31	1,60



20c)  $h_k = \sin \frac{1}{2}t = 0,5 \Rightarrow \sin \frac{1}{2}t = \sin \frac{1}{4}\pi \vee \sin \frac{1}{2}t = \sin \frac{5}{6}\pi \Rightarrow \frac{1}{2}t = \frac{1}{6}\pi \vee \frac{1}{2}t = \frac{5}{6}\pi$

$\Rightarrow t = \frac{1}{3}\pi \vee t = \frac{5}{3}\pi \rightarrow$

$h_v(\frac{1}{3}\pi) = 2 \sin \frac{1}{2}(\frac{1}{3}\pi + 5) \simeq 0,24 \text{ m} \vee h_v(\frac{5}{3}\pi) = 2 \sin \frac{1}{2}(\frac{5}{3}\pi + 5) \simeq -1,84 \text{ m}$

Je kunt het ook aflezen, dit is echter minder nauwkeurig...

21a&b)

$$l : \left\{ \begin{array}{l} (2;0) \\ (3;1) \end{array} \right\} \Rightarrow r.c. = \frac{\Delta y}{\Delta x} = \frac{1-0}{3-2} = \frac{1}{1} = 1$$

$$\left. \begin{array}{l} y = x + b \\ (2;0) \end{array} \right\} \Rightarrow 0 = 2 + b \Rightarrow b = -2$$

$$\xrightarrow{b=-2 \text{ en } r.c.=1} l : y = x - 2$$

21d)

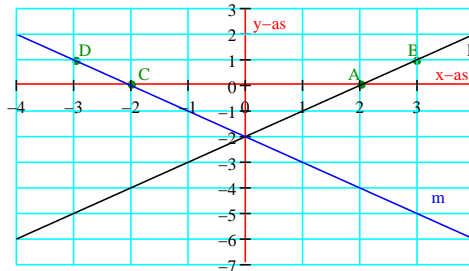
$$C : (-2; 0)$$

$$B : (-3; 1)$$

21e)

$$m : y = -x - 2$$

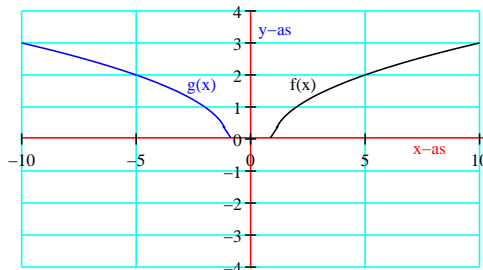
plaatje bij Som 21)



22a)  $f(x) = \sqrt{x-1} D_f : x-1 \geq 0 \Rightarrow x \geq 1$

$x$	1	2	5	10
$f(x)$	0	1	2	3

plaatje bij Som 22)



22b)  $g(x) = \sqrt{-x-1}$

23)  $Top_{g(x)} : (2; -2) \Rightarrow Top_{f(x)} : (-2; -2)$

I:  $f(x) = x^2 - 2x \xrightarrow{\text{Snijpunt met } x\text{-as}} x^2 - 2x = 0 \Rightarrow x(x-2) = 0 \Rightarrow x = 0 \vee x = 2 \xrightarrow{\text{Top}} (1; -1)$

II:  $f(x) = x^2 + 2x \xrightarrow{\text{Snijpunt met } x\text{-as}} x^2 + 2x = 0 \Rightarrow x(x+2) = 0 \Rightarrow x = 0 \vee x = -2 \xrightarrow{\text{Top}} (-1; -1)$

Dus geen van beiden.

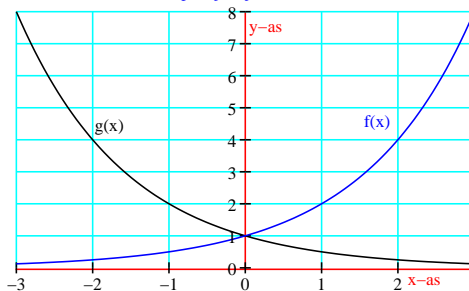
24)

$$\left. \begin{array}{l} g(x) = \left(\frac{1}{2}\right)^x = (2^{-1})^x = 2^{-x} \\ f(x) = 2^{-x} \end{array} \right\} \Rightarrow$$

$$\Rightarrow f(x) = g(x)$$

$x$	-1	0	1	2	3
$f(x)$	$\frac{1}{2}$	1	2	4	8
$g(x)$	2	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$

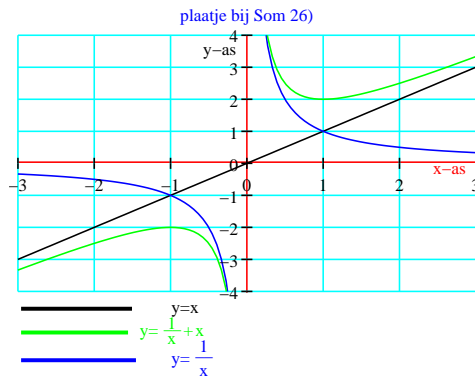
plaatje bij Som 24)



## KERN 3 COMBINEREN

25) Kijk in't Antwoordenboek.....

26a)



26b) x-as en de y-as

$x$		-3	-2	-1	0	1	2	3	10
$f$		-3	-2	-1	0	1	2	3	10
$g$		$-\frac{1}{3}$	$-\frac{1}{2}$	-1	-	1	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{10}$
$f+g$		$-3\frac{1}{3}$	$-2\frac{1}{2}$	-2	-	2	$2\frac{1}{2}$	$3\frac{1}{3}$	$10\frac{1}{10}$

26c)

$x$		$-\frac{1}{2}$	$-\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{10}$
$f$		$-\frac{1}{2}$	$-\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{10}$
$g$		-2	-4	2	4	10
$f+g$		$-2\frac{1}{2}$	$-4\frac{1}{4}$	$2\frac{1}{2}$	$4\frac{1}{4}$	$10\frac{1}{10}$

26d) De lijn  $y = x$  en de  $y=as$

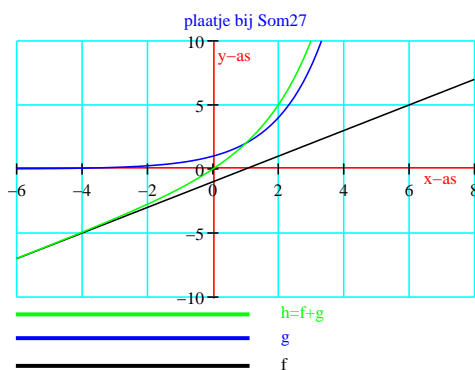
27)  $f(x) = x - 1$   $g(x) = 2^x$   $h = f + g$

$x$		-4	-2	0	1	2	3	4
$f$		-5	-3	-1	0	1	1	3
$g$		$\frac{1}{16}$	$\frac{1}{4}$	1	2	4	8	16
$f+g$		$-4\frac{5}{16}$	$-2\frac{3}{4}$	0	2	5	10	19

27c)  $y = x - 1$  ← *Asymptoot van h*

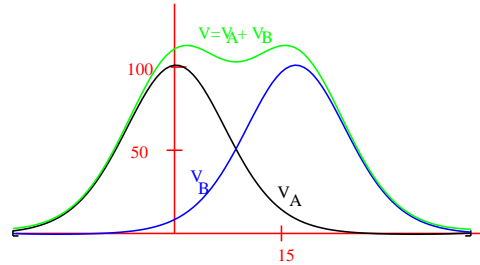
Als  $x$  heel groot negatief wordt bijvoorbeeld  $x = -1000 \Rightarrow 2^{-1000} \simeq 0$

Bij  $f(x) = x + 1$  tel je dan een heel klein getal op



28)

plaatje bij Som 28)



$10 \iff 0,4 \text{ cm}$

$25 \iff 1,0 \text{ cm}$

- $x = 2 \quad 3,6 \text{ cm} \quad \Rightarrow \quad 3,6 \cdot 25 = 90$
- $x = 4 \quad 2,9 \text{ cm} \quad \Rightarrow \quad 2,9 \cdot 25 \approx 72,5$
- $x = 6 \quad 2,0 \text{ cm} \quad \Rightarrow \quad 2,0 \cdot 25 = 50$
- $x = 8 \quad 1,3 \text{ cm} \quad \Rightarrow \quad 1,3 \cdot 25 \approx 32,3$
- $x = 10 \quad 0,8 \text{ cm} \quad \Rightarrow \quad 0,8 \cdot 25 = 20$
- $x = 12 \quad 0,5 \text{ cm} \quad \Rightarrow \quad 0,5 \cdot 25 = 12,5$
- $x = 14 \quad 0,35 \text{ cm} \quad \Rightarrow \quad 0,35 \cdot 25 = 9$
- $x = 15 \quad 0,35 \text{ cm} \quad \Rightarrow \quad 0,35 \cdot 25 = 9$

V	-4	-2	0	2	4	6	8	10	12	14	15
$V_A$	72,5	90	100	90	72,5	50	32,5	20	12,5	9	9
$V_B$	9	9	12,5	20	32,5	50	72,5	90	100	90	72,5
$V = V_A + V_B$	89,5	99	112,5	110	105	100	105	110	112,5	99	89,5

28c) Bij  $x = 6 \rightarrow V = 100$  Zie Antwoordenboek.....

30) 1 eenheid  $\iff 0,8 \text{ cm} \Rightarrow \frac{1}{0,8} \text{ eenheid} \iff 1 \text{ cm}$

Bijvoorbeeld: 4<sup>de</sup> eenheid op  $q - as$  Verskil van 0,9cm  $\rightarrow \frac{0,9}{0,8} \approx 1,125 \text{ cm}$

31a)  $f(x) = 20x - x^2 \quad g(x) = 56 + 2x$

$f(x) = 0 \Rightarrow 20x - x^2 = 0 \Rightarrow x(20 - x) = 0 \Rightarrow x = 0 \vee x = 20$

x	-1	0	1	2	10	18	19	20	21
$f(x) = 20x - x^2$	-28	0	19	36	100	36	19	0	-21
$g(x) = 56 + 2x$	54	56	58	60	76	92	94	96	98
$f(x) - g(x)$	-75	-56	-39	-24	24	-56	-75	-96	-119

31b)  $20x - x^2 = 56 + 2x \xrightarrow{-2x} 18x - x^2 = 56 \Rightarrow 18x - x^2 - 56 = 0 \Rightarrow (x - 4)(x - 14) = 0 \Rightarrow x = 4 \vee x = 14$

$f(4) = 20 \cdot 4 - 4^2 = 64$  Het snijpunt  $\rightarrow (4; 64)$

$f(14) = 20 \cdot 14 - 14^2 = 84$  Het snijpunt  $\rightarrow (14; 84)$

Produkt		Som (-18)		$-x^2 + 18x - 56 = 0$
$7 \cdot 8 = 56$		$7 + 8 = 15$		fout
$4 \cdot 14 = 56$		$4 + 14 = 18$		fout
$-4 \cdot (-14) = 56$		$-4 - 14 = 18$		Goed

$f(x) \quad << \quad = \quad > \quad > \quad > \quad = \quad <<< \quad g(x)$   
4 14



**31d)** (4;0) en (14;0)

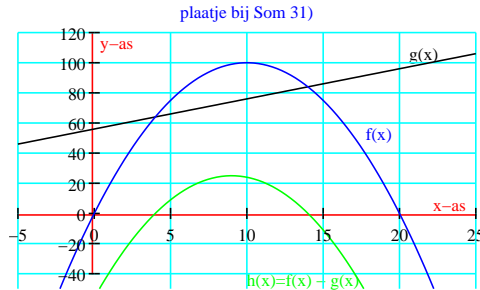
**31e)**  $h(x) = 20x - x^2 - 56 - 2x \Rightarrow h(x) = -x^2 + 18x - 56$

Symmetrie-as ligt precies tussen (4;0) en (14;0)  $\Rightarrow y = \frac{14-4}{2} = 9$

**31f)**  $h_{max} = h(9) = -9^2 + 18 \cdot 9 - 56 = 25$

**31g)** Zie grafiek

**31h)** Zie 31e)



**32a)**

$$\left. \begin{matrix} y_1 = 3^x \\ y_2 = 3^{-x} \end{matrix} \right\} \Rightarrow y_1 + y_2 = 3^x + 3^{-x} \xrightarrow{x=0} 3^0 + 3^{-0} = 1 + 1 = 2 \Rightarrow \begin{cases} A \text{ hoort bij } y_1 + y_2 \\ B \text{ hoort bij } y_1 - y_2 \end{cases}$$

**32b)**

$$y_1 + y_2 \xrightarrow{\text{Zie hierboven} \uparrow} \\ y_1 - y_2 = 3^x - 3^{-x}$$

**33)**  $k(x) = \cos(x) - \sin(x)$

**33a)**  $k(x)$  is een verschilfunctie van twee periodieke functies  $\xrightarrow{\text{Periode is}} 2\pi$

**33b)**  $x = \frac{1}{4}\pi$  en  $x = 1\frac{1}{4}\pi$

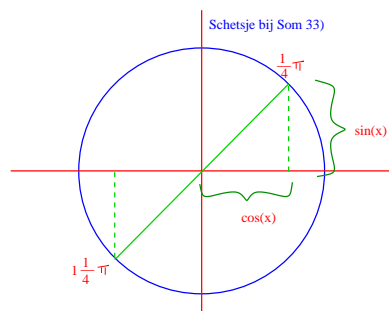
**33c)**  $k(x) = p \sin(x - q)$

$k(0) = p \sin(-q) = 1$

$k(\frac{1}{4}\pi) = p \sin(\frac{1}{4}\pi - q) = 0 \Rightarrow \sin(\frac{1}{4}\pi - q) = 0 = \sin(0 + k\pi) \Rightarrow$

$\Rightarrow \frac{1}{4}\pi - q = 0 + k\pi \Rightarrow q = \frac{1}{4}\pi \vee q = 1\frac{1}{4}\pi \leftarrow \frac{1}{4}\pi \text{ naar rechts verschoven}$

$p \sin(-1\frac{1}{4}\pi) = 1 \Rightarrow p \cdot \frac{1}{2}\sqrt{2} = 1 \Rightarrow p = \frac{1}{\frac{1}{2}\sqrt{2}} = \frac{2}{\sqrt{2}} = \frac{2\sqrt{2}}{\sqrt{2} \cdot \sqrt{2}} = \frac{2\sqrt{2}}{2} = \sqrt{2} \approx 1,41$



## KERN 4 SCHAKELN

**34a)**  $u = \sqrt{(b-7)}$

$b \xrightarrow{-7} *** \xrightarrow{\sqrt{\quad}} u$

**34b)**  $u = {}^2 \log(5b)$

$b \xrightarrow{\times 5} *** \xrightarrow{{}^2 \log ***} u$

**34c)**  $y = (2x+3)^4$

$x \xrightarrow{\times 2} *** \xrightarrow{+3} *** \xrightarrow{(***)^4} y$

**34d)**  $y = 2^{(2t-1)}$

$t \xrightarrow{\times 2} *** \xrightarrow{-1} *** \xrightarrow{2^{***}} y$

**35)**  $t = b^2 \quad u = t + 16$

**35a)**  $b = 3 \Rightarrow t = 3^2 = 9$

**35b)**  $b = 3 \Rightarrow t = 9 \Rightarrow u = 9 + 16 = 25$

**35c)**  $u = b^2 + 16$

**35d)**  $t_1 = b^2 \quad t_2 = t_1 + 16 \quad u = \sqrt{t_2}$

$b = 3 \Rightarrow t_1 = 3^2 = 9 \Rightarrow t_2 = 9 + 16 = 25 \Rightarrow u = \sqrt{25} \Rightarrow u = 5$

**35e)**  $b = 0 \Rightarrow t_1 = 0 \Rightarrow t_2 = 16 \Rightarrow u = \sqrt{16} \Rightarrow u = 4$

$b = 8 \Rightarrow t_1 = 8^2 = 64 \Rightarrow t_2 = 64 + 16 = 80 \Rightarrow u = \sqrt{80} = \sqrt{16 \cdot 5} = \sqrt{16} \cdot \sqrt{5} \Rightarrow u = 4\sqrt{5} \simeq 8,9$

**35f)**

$$\left. \begin{array}{l} u = \sqrt{t_2} \\ t_2 = t_1 + 16 \end{array} \right\} \Rightarrow \left. \begin{array}{l} u = \sqrt{t_1 + 16} \\ t_1 = b^2 \end{array} \right\} \Rightarrow u = \sqrt{b^2 + 16}$$

**36a)**  $\left. \begin{array}{l} y = \sin(t) \\ t = x - 8 \end{array} \right\} \Rightarrow y = \sin(x - 8)$

**36b)**  $\left. \begin{array}{l} y = t^{10} \\ t = x + 2 \end{array} \right\} \Rightarrow y = (x + 2)^{10}$

**36c)**  $\left. \begin{array}{l} t_2 = t_1 - 5 \\ t_1 = x^3 \end{array} \right\} \Rightarrow \left. \begin{array}{l} t_2 = x^3 - 5 \\ y = \sqrt{t_2} \end{array} \right\} \Rightarrow y = \sqrt{(x^3 - 5)}$

**36d)**  $\left. \begin{array}{l} t_1 = \sqrt{x} \\ t_2 = t_1 - 5 \end{array} \right\} \Rightarrow \left. \begin{array}{l} t_2 = \sqrt{x} - 5 \\ y = t_2^3 \end{array} \right\} \Rightarrow y = (\sqrt{x} - 5)^3$

**37a)**  $\left. \begin{array}{l} t = b - 6 \\ u = \sqrt{t} \end{array} \right\} \Rightarrow u = \sqrt{(b - 6)}$

**37b)**  $\left. \begin{array}{l} t_1 = 4b \\ t_2 = t_1 + 7 \end{array} \right\} \Rightarrow \left. \begin{array}{l} t_2 = 4b + 7 \\ u = 3^{t_2} \end{array} \right\} \Rightarrow u = 3^{(4b+7)}$

**38a)**  $b \xrightarrow{\times 3} t_1 \xrightarrow{+1} t_2 \xrightarrow{{}^2 \log **} u \quad \left\{ \begin{array}{l} t_1 = 3b \\ t_2 = t_1 + 1 \\ u = {}^2 \log t_2 \end{array} \right.$

**38b)**  $x \xrightarrow{-2} t_1 \xrightarrow{\sin **} t_2 \xrightarrow{\times 5} y \quad \left\{ \begin{array}{l} t_1 = x - 2 \\ t_2 = \sin t_1 \\ y = 5 \cdot t_2 \end{array} \right.$

**39)**  $c = \frac{p+10}{10}$

**39a)**  $c = \frac{57+10}{10} = 6,7$

**39b)**  $d = 0,8 \cdot c + 2$

$p = 57 \Rightarrow d = 0,8 \cdot 6,7 + 2 \simeq 7,4$

**39c)**  $d = 0,8 \cdot \frac{p+10}{10} + 2 \Rightarrow d = 2 + (p+10) \frac{0,8}{10} = 2 + 0,08p + 0,8 \Rightarrow d = 0,08p + 2,8$

**40a)**

$$\left. \begin{array}{l} f(x) = 2x - 1 \Rightarrow t = 2x - 1 \\ g(x) = 2^x \Rightarrow y = 2^t \end{array} \right\} \Rightarrow y = 2^{2x-1}$$

**40b)**

$$\left. \begin{array}{l} f(x) = x^2 + 10 \Rightarrow t = x^2 + 10 \\ g(x) = \frac{1}{x} \Rightarrow y = \frac{1}{t} \end{array} \right\} \Rightarrow y = \frac{1}{x^2+10}$$

**40c)**

$$\left. \begin{array}{l} f(x) = \frac{1}{x} \Rightarrow t = \frac{1}{x} \\ g(x) = x^2 + 10 \Rightarrow y = t^2 + 10 \end{array} \right\} \Rightarrow y = \left(\frac{1}{x}\right)^2 + 10 \Rightarrow y = \frac{1}{x^2} + 10$$

41a)  $f(x) = 3x + 1$   $g(x) = 4^x$

41b)  $f(x) = 2x + 1$   $g(x) = x^8$

41c)  $f(x) = x^3 - 3x$   $g(x) = \log x$

41d)  $f(x) = \sin \pi x$   $g(x) = x^2$

42a) 100 ml per minuut  $\Rightarrow$

$100 \cdot 8 = 800$  ml in 8 minuten

42b) Oppervlakte bodem= $\Rightarrow$

$40 \text{ cm} \rightarrow \frac{800}{40} = 20 \text{ cm}$

42c)  $V = 100h$

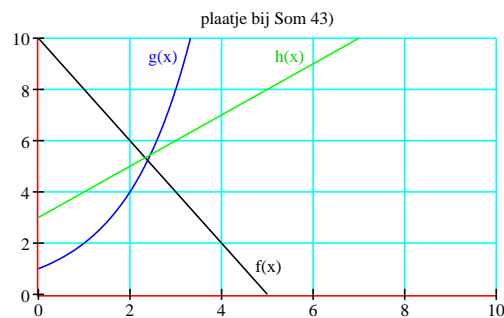
42d)  $h = \frac{V}{40}$

42e)  $b \xrightarrow{\times 100} *** \xrightarrow{\div 40} h$

42f)  $h = \frac{100t}{40} = 2\frac{1}{2}t$

43)  $f(x) = 10 - 2x$   $g(x) = 2^x$   $h(x) = x + 3$  Domein :  $[0; 5]$

43a)



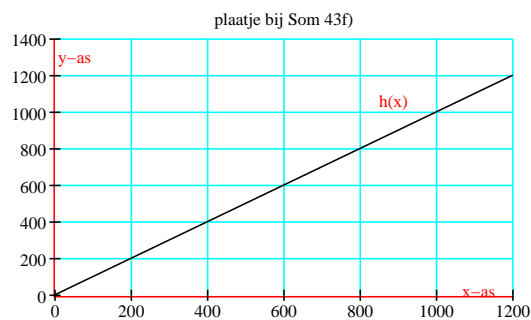
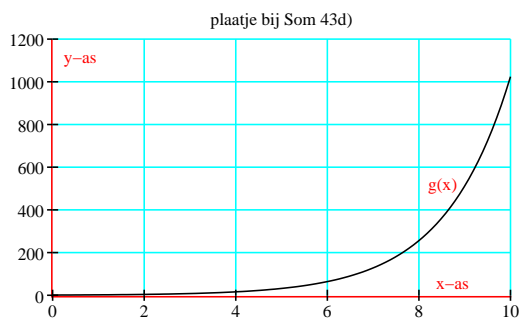
43b)  $B_f = [0; 10]$

43c)

Neen  $\rightarrow$  door  $f$  wordt  $-2$  niet bereikt

ja  $\rightarrow$  door  $f$  wordt  $2$  wel bereikt

43d)  $D_g : [0; 10] = B_f$



43e)  $B_g = [1; 1024]$

43f)  $D_h = [1; 1024] = B_g$

43g&h)  $B_h = [3; 1027]$

43i)  $y = 2^{(10-2x)} + 3$

$$\left. \begin{array}{l} f(x) = 10 - 2x \Rightarrow t_1 = 10 - 2x \\ g(x) = 2^x \Rightarrow t_2 = 2^{t_1} \end{array} \right\} \Rightarrow \left. \begin{array}{l} t_2 = 2^{10-2x} \\ h(x) = x + 3 \Rightarrow y = t_2 + 3 \end{array} \right\} \Rightarrow y = 2^{10-2x} + 3$$

44a)  $t = 1$

44b)  $x^2 = 4 \Rightarrow x = 2 \vee x = -2$

44c)

$$\left. \begin{array}{l} t = 5 - x^2 \\ y = \log t \end{array} \right\} \Rightarrow y = \log(5 - x^2) \xrightarrow{\text{Asymptoten}} 5 - x^2 = 0 \Rightarrow x^2 = 5 \Rightarrow x = \sqrt{5} \vee x = -\sqrt{5}$$



## GRAFISCHE REKENMACHINE

**G3a)**  $f(x) = {}^2\log(-x^2 + 8x - 12) \xrightarrow{\text{intypen}} \frac{\log(-x^2 + 8x - 12)}{\log 2}$

**G3b)**  $-x^2 + 8x - 12 = 0 \Rightarrow x^2 - 8x + 12 = 0 \Rightarrow (x-6)(x-2) = 0 \Rightarrow x = 6 \vee x = 2$

**G3c)**  $x_{\text{sym}} = 4 \Rightarrow -4^2 + 8 \cdot 4 - 12 = 4 \xrightarrow{^2\log} \log 4 = 2$

**G3d)**  ${}^2\log(-x^2 + 8x - 12) = 0 \Rightarrow -x^2 + 8x - 12 = 2^0 = 1 \Rightarrow x^2 - 8x + 13 = 0 \xrightarrow{\text{ABC-formule}}$   
 $\Rightarrow x = \frac{8 \pm \sqrt{64 - 4 \cdot 13}}{2} \Rightarrow x = \frac{8 \pm \sqrt{12}}{2} \Rightarrow x = \frac{8 \pm \sqrt{4 \cdot 3}}{2} \Rightarrow x = \frac{8 \pm 2\sqrt{3}}{2} \Rightarrow x = 4 - \sqrt{3} \vee x = 4 + \sqrt{3}$   
 $\Rightarrow x \simeq 2,27 \vee x \simeq 5,73$

$x_B - x_A = 4 + \sqrt{3} - (4 - \sqrt{3}) = 2\sqrt{3} \simeq 3,46$

**G3e)**  $A'B' = 2 \ x_{\text{sym}} = 4 \Rightarrow \left. \begin{array}{l} x_{B'} = 5 \\ x_{A'} = 3 \end{array} \right\}$

$g(x) = {}^2\log(-x^2 + 8x - 12) + p \ (5; 0) \in g$

$0 = {}^2\log(-5^2 + 8 \cdot 5 - 12) + p \Rightarrow 0 = {}^2\log 3 + p \Rightarrow p = -{}^2\log 3 \simeq -1,58$

**G4a)**  $|AC| = \sqrt{1200^2 + 500^2} = 1300$

$1300 \cdot 8 = \text{Hfl } 10.400,-$

**G4b)**  $1200 \cdot 5 + 500 \cdot 8 = \text{Hfl } 10.000,-$

**G4c)**  $BD = 1200 - x$

$|BC| = \sqrt{(1200 - x)^2 + 500^2}$

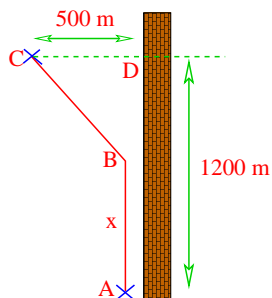
$K_1 = 5x$

$K_2 = 8 \cdot \sqrt{(1200 - x)^2 + 500^2}$

**G4d)**  $K_1 + K_2 = 5x + 8 \cdot \sqrt{x^2 - 2400x + 1690000}$

$x \simeq 800$

$y \simeq \text{Hfl } 9122,60$



## DOOR WERKING

**D1a)** Zomer en Wintertijd

**D1b)**  $y_{na} = (y_{voor} + 1)$

**D1c)** 1 omhoog schuiven

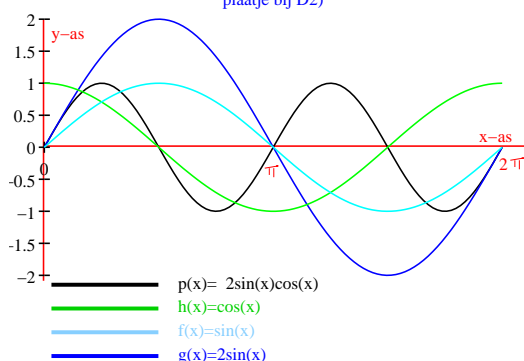
**D2a)** Vertikaal met 2 vermenigvuldigen (de y-waarde dus met twee vermenigvuldigen)

**D2b)**  $\frac{1}{2}\pi$  naar links verschuiven of  $1\frac{1}{2}\pi$  naar rechts verschuiven

**D2c)**  $p(x) = g(x) \cdot h(x) = 2 \cdot \sin(x) \cdot \cos(x)$

$x$	0	$\frac{1}{6}\pi$	$\frac{1}{4}\pi$	$\frac{1}{3}\pi$	$\frac{1}{2}\pi$	$\frac{2}{3}\pi$	$\frac{3}{4}\pi$	$\frac{5}{6}\pi$	$\pi$	$1\frac{1}{6}\pi$	$1\frac{1}{4}\pi$	$1\frac{1}{3}\pi$	$1\frac{1}{2}\pi$
$f(x) = \sin x$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1
$g(x) = 2 \sin x$	0	1	$\sqrt{2}$	$\sqrt{3}$	2	$\sqrt{3}$	$\sqrt{2}$	1	0	-1	$-\sqrt{2}$	$-\sqrt{3}$	-2
$h(x) = \cos x$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	0
$p(x)$	0	$\frac{1}{2}\sqrt{3}$	1	$\frac{1}{2}\sqrt{3}$	0	$-\frac{1}{2}\sqrt{3}$	-1	$-\frac{1}{2}\sqrt{3}$	0	$\frac{1}{2}\sqrt{3}$	+1	$\frac{1}{2}\sqrt{3}$	0

plaatje bij D2)



**D2d)**

Nulpunten :  $(0; 0), (\frac{1}{2}\pi; 0), (\pi; 0)$  etc etc  $\xrightarrow{\text{Dus } k \in \mathbb{Z}} (\frac{1}{2}k\pi; 0)$

$k \in \mathbb{Z} \xleftarrow{\text{d.w.z. } k \text{ is geheel getal}}$

Maxima :  $(\frac{1}{4}\pi; 1), (1\frac{1}{4}\pi; 1), (2\frac{1}{4}\pi; 1)$  etc etc  $\xrightarrow{\text{Dus } k \in \mathbb{Z}} (\frac{1}{4}\pi + k\pi; 1)$

$k \in \mathbb{Z} \xleftarrow{\text{d.w.z. } k \text{ is geheel getal}}$

Minima :  $(\frac{3}{4}\pi; -1), (1\frac{3}{4}\pi; -1), (2\frac{3}{4}\pi; -1)$  etc etc  $\xrightarrow{\text{Dus } k \in \mathbb{Z}} (\frac{3}{4}\pi + k\pi; -1)$

$k \in \mathbb{Z} \xleftarrow{\text{d.w.z. } k \text{ is geheel getal}}$

**D2e)** Vermenigvuldigen met een  $\frac{1}{2}$  t.o.v. de y-as

$$\left. \begin{array}{l} (\frac{1}{2}\pi; 1) \xrightarrow{\text{Wordt}} (\frac{1}{4}\pi; 1) \\ (\pi; 0) \xrightarrow{\text{wordt}} (\frac{1}{2}\pi; 0) \end{array} \right\} \xrightarrow{\text{Formule}} p(x) = \sin(2x) \Rightarrow p(x) = 2 \cdot \sin(x) \cdot \cos(x)$$

**D3a)**

**A:**  $(-4; 0) \xrightarrow{y_{waarde}=0 \dots \text{want}} 0 \cdot \text{getal} = 0$

**B:**  $(-1; 3) \xrightarrow{y_{waarde}=3}$  de y-waarde van het punt met x-waarde= -1, liggend op p is 1 en  $3 \cdot 1 = 3$

**C:**  $(1; 5) \xrightarrow{y_{waarde}=5}$  de y-waarde van het punt met x-waarde= 1, liggend op p is 1 en  $5 \cdot 1 = 5$

**D3b)**

$(0; 0) \xrightarrow{\text{Want}} 0 \cdot 4 = 0$

$(-3; 9) \xrightarrow{\text{Want}} 9 \cdot 1 = 9$

**Of:**  $x^2 = x^2(x+4) \Rightarrow x^2 \cdot 1 = x^2(x+4) \Rightarrow x = 0 \vee x = -3$

$$l : y = x + 6 \rightarrow (-6;0) (-1;5) (1;7)$$

$$p : y = x^2 \rightarrow (0;0)$$

$$l : y = x + 2 \rightarrow (-2;0) (-1;1) (1;3)$$

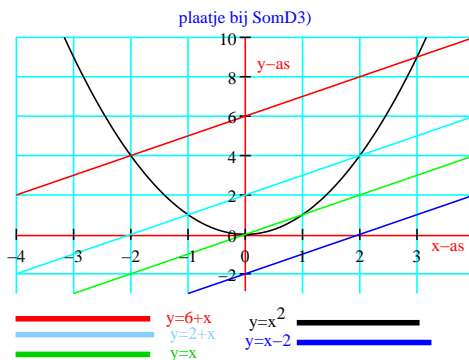
$$p : y = x^2 \rightarrow (0;0)$$

$$l : y = x \rightarrow (-1;-1) (0;0) (1;1)$$

$$p : y = x^2 \rightarrow (0;0)$$

$$l : y = x - 2 \rightarrow (1;-1) (-1;-3) (2;0)$$

$$p : y = x^2 \rightarrow (0;0)$$



**D3d)**  $a = 4 \Rightarrow y = x^2(x + 4)$

$$-\frac{1}{2}x^3 = x^2(x + 4) \xrightarrow{\div 4 \dots \text{want } x^2=0 \text{ levert je niet de Top op}} -\frac{1}{2}x = x + 4 \Rightarrow$$

$$\Rightarrow 1\frac{1}{2}x = -4 \Rightarrow x = -4 \cdot \frac{2}{3} \Rightarrow x = -\frac{8}{3} = -2\frac{2}{3} \rightarrow$$

$$y = -\frac{1}{2} \left(-\frac{8}{3}\right)^3 \Rightarrow y = -\frac{1}{2} \cdot -\frac{512}{27} = \frac{256}{27} \Rightarrow y = 9\frac{13}{27}$$

$$\Rightarrow \text{Top} : \left(-2\frac{2}{3}; 9\frac{13}{27}\right)$$

**Of:**  $-\frac{1}{2}x^3 = x^2(x + 4) \Rightarrow -\frac{1}{2}x^3 = x^3 + 4x^2 \Rightarrow 1\frac{1}{2}x^3 + 4x^2 = 0 \Rightarrow x^2(1\frac{1}{2}x + 4) = 0 \Rightarrow$

$$\Rightarrow x^2 = 0 \vee 1\frac{1}{2}x + 4 = 0 \rightarrow \text{etc etc}$$

**D3e)**  $a = -4 \Rightarrow y = x^2(x - 4)$

$$1\frac{1}{2}x^3 = x^2(x - 4) \xrightarrow{\div x^2 \dots \dots \dots x^2=0 \text{ levert geen Top zie grafiek}} -\frac{1}{2}x = x - 4 \Rightarrow 1\frac{1}{2}x - 4 \Rightarrow x = \frac{8}{3} = 2\frac{1}{3} \rightarrow$$

$$\rightarrow y = -\frac{1}{2} \cdot \left(\frac{8}{3}\right)^3 = -9\frac{13}{27}$$

$$\Rightarrow \text{Top} : \left(2\frac{1}{3}; -9\frac{13}{27}\right)$$

