123

UITWERKINGEN VOOR HET VWO
A1B1 DEEL1

Hoofdstuk 4
LOGARITMISCHE FUNCTIES

KERN 1
LOGARITMEN

1) $H = 10 \cdot 2^t$ waarbij t in weken
1a) Verdubbeling na 1 week
8 keer zoveel na 3 weken
1b)
$t = 1 \Rightarrow H = 10 \cdot 2^1 = 30$
$t = 3 \Rightarrow H = 10 \cdot 2^3 = 10 \cdot 8 = 80$

2a)
$^2\log = 1 \xrightarrow{Want} 2^1 = 2$
$^2\log = 16 \xrightarrow{Want} 2^4 = 16$
$^2\log = 64 \xrightarrow{Want} 2^6 = 64$
2b) $2^t = 7 \Rightarrow t = ^2\log 7$

3a) $f(t) = 3^t \; t \in [-2; 2]$

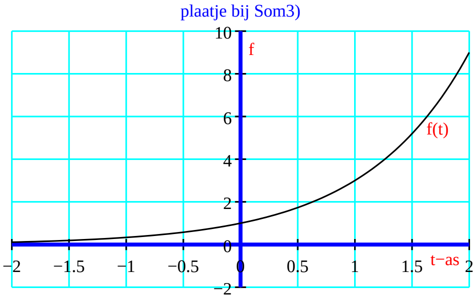
t	-2	-1	0	1	2
$f(t)$	$\frac{1}{9}$	$\frac{1}{3}$	1	3	9

3b)
$3^t = 6 \Rightarrow t \simeq 1,6$
$3^t = 1 \Rightarrow t = 0$
$3^t = \frac{1}{2} \Rightarrow t \simeq -0,6$
2c)
$3^{1,4} \simeq 5,8$
$3^0 = 1$
$3^{-0,6} \simeq 0,52$

1c) 2,3 weken
1d)
$t = 2,3 \Rightarrow H = 10 \cdot 2^{2,3} \simeq 49,2$
$t = 2,4 \Rightarrow H = 10 \cdot 2^{2,4} \simeq 52,8$
1e) neen

2c)
$2^2 = 4$
$2^3 = 8$
$2^{2,5} \simeq 5,66$
$2^{2,6} \simeq 6,06$
$2^{2,7} \simeq 6,50$
$$\left.\begin{array}{l} 2^{2,8} \simeq 6,96 \\ 2^{2,9} \simeq 7,46 \end{array}\right\} \Rightarrow t \simeq 2,8$$

plaatje bij Som3)



[1]*Deze samenvatting mag niet massaal op kosten van Schaersvoorde worden Uitgeprint!!!*
[2]*werd gemaakt onder LinuX met LaTeX en LyX*
[3]*Typ&andere fouten&blunders graag Melden!*

4a) $\left(\frac{1}{2}\right)^t = 4 \Rightarrow t = -2$
4b) ${}^{\frac{1}{2}}\log\frac{1}{4} = 2 \xrightarrow{Want} \left(\frac{1}{2}\right)^2 = \frac{1^2}{2^2} = \frac{1}{4}$
4b) ${}^{\frac{1}{2}}\log 1 = 0 \xrightarrow{Want} \left(\frac{1}{2}\right)^0 = 1$

4d) $\left(\frac{1}{2}\right)^3 = \frac{1}{8} = 0,125$
$\left(\frac{1}{2}\right)^4 = \frac{1}{16} = 0,0625$
Dus ${}^{\frac{1}{2}}\log 0,1$ ligt tussen 3 en 4

5) $H = 1,8^t$, t in dagen, H in Miljoenen
5a) $t = 0 \Rightarrow H = 1,8^0 = 1$
$$\left.\begin{array}{l} t = 1,1 \Rightarrow H \simeq 1,91 \\ t = 1,2 \Rightarrow H \simeq 2,02 \\ t = 1,3 \Rightarrow H \simeq 2,15 \end{array}\right\} \Rightarrow t = 1,2\,dagen$$
$$\left.\begin{array}{l} t = 2,6 \Rightarrow H \simeq 4,61 \\ t = 2,7 \Rightarrow H \simeq 4,89 \\ t = 2,8 \Rightarrow H \simeq 5,19 \end{array}\right\} \Rightarrow t = 2,7\,dagen$$

5b) $1,8^{1,2} \simeq 2 \quad 1,8^{2,7} \simeq 5$
5c) ${}^{1,8}\log 2 \simeq 1,2 \quad {}^{1,8}\log 5 \simeq 2,7$
5d)
$$\left.\begin{array}{l} t = 3,8 \Rightarrow H = 1,8^{3,8} \simeq 9,33 \\ t = 3,9 \Rightarrow H = 1,8^{3,9} \simeq 9,90 \\ t = 4 \Rightarrow H = 1,8^4 \simeq 10,50 \end{array}\right\} \Rightarrow t = 3,9$$
5e) $1,2 + 2,7 = 3,9$

6a) $4^3 = 64$
6b) $4^t = 64 \Rightarrow t = {}^4\log 64 = 3$
6c) ${}^4\log 64 = 3$

6d) $2^6 = 64 \Rightarrow {}^2\log 64 = 6$
6e) ${}^8\log 64 = 2 \xrightarrow{want} 8^2 = 64$

7a) ${}^2\log 128 = 7 \xrightarrow{Want} 2^7 = 128$
7b) ${}^3\log 81 = 4 \xrightarrow{Want} 2^4 = 81$
7c) ${}^{\frac{1}{5}}\log 5 = -1 \xrightarrow{Want} \left(\frac{1}{5}\right)^{-1} = \left(5^{-1}\right)^{-1} = 5$
7d) ${}^3\log 9 = 2 \xrightarrow{Want} 3^2 = 9$

7e) ${}^{10}\log 0,001 = -3 \xrightarrow{Want} 10^{-3} = 0,001$
7f) ${}^5\log 125 = 3 \xrightarrow{Want} 5^3 = 125$
7g) ${}^4\log 0,25 = -1 \xrightarrow{Want} 4^{-1} = \frac{1}{4} = 0,25$
7h) ${}^6\log 36 = 2 \xrightarrow{Want} 6^2 = 36$

8a) ${}^{10}\log 0,1 = -1 \xrightarrow{Want} 10^{-1} = 0,1$
8b) ${}^{0,1}\log 0,01 = 2 \xrightarrow{Want} 0,1^2 = 0,01$
8c) ${}^2\log\frac{1}{4} = -2 \xrightarrow{Want} 2^{-2} = \frac{1}{4}$
8d) ${}^3\log\frac{1}{27} = -3 \xrightarrow{Want} 3^{-3} = \frac{1}{3^3} = \frac{1}{27}$

8e) ${}^{\frac{1}{5}}\log\frac{1}{125} = 3 \xrightarrow{Want} \left(\frac{1}{5}\right)^3 = \frac{1}{125}$
8f) ${}^3\log\sqrt{27} = {}^3\log\sqrt{3^3} = {}^3\log 3^{\frac{3}{2}} = \frac{3}{2} \xrightarrow{Want}$
$\xrightarrow{Want} 3^{\frac{3}{2}} = \sqrt{3^3} = \sqrt{27}$

9a) ${}^x\log 8 = 3 \Rightarrow x^3 = 8 \Rightarrow x = 2$
9b) ${}^3\log x = 4 \Rightarrow x = 3^4 = 81$
9c) ${}^x\log 3 = 3 \Rightarrow x^3 = 3 \Rightarrow x = 3^{\frac{1}{2}} = \sqrt[3]{3}$
9d) ${}^x\log 3 = -1 \Rightarrow x^{-1} = 3 \Rightarrow \frac{1}{x} = 3 \Rightarrow x = \frac{1}{3}$

9e) ${}^x\log 9 = 2 \Rightarrow x^2 = 9 \Rightarrow x = 3$
9f) ${}^x\log 9 = -2 \Rightarrow x^{-2} = 9 \Rightarrow \frac{1}{x^2} = 9 \Rightarrow$
$\Rightarrow x^2 = \frac{1}{9} \Rightarrow x = \frac{1}{3}$

10a) $2^t = 64 \Rightarrow t = 6$
10b) $2^{t+3} = 64 \Rightarrow t + 3 = 6 \Rightarrow t = 3$
10c) $2^{3t} = 64 \Rightarrow 3t = 6 \Rightarrow t = 2$

10d) ${}^2\log x = 5 \Rightarrow x = 2^5 = 32$
10e) ${}^2\log(x+2) = 5 \Rightarrow x + 2 = 32 \Rightarrow x = 30$
10f) ${}^2\log 8x = 5 \Rightarrow 8x = 32 \Rightarrow x = 4$

11a) $5^t = 8 \Rightarrow t = {}^5\log 8 \simeq 1,3$
$\left(5^{1,2} \simeq 6,90\ 5^{1,3} \simeq 8,1\right)$
11b) $5^{t-1} = 8 \Rightarrow t - 1 \simeq 1,3 \Rightarrow t \simeq 2,3$
11c) $\left(\frac{1}{2}\right)^t = 0,1 \Rightarrow t = {}^{\frac{1}{2}}\log 0,1 \simeq 3,3$

$\left(\left(\frac{1}{2}\right)^{3,3} \simeq 0,102\right)$
11d) $\left(\frac{1}{2}\right)^{2t-1} = 0,1 \Rightarrow 2t - 1 = {}^{\frac{1}{2}}\log 0,1$
$\Rightarrow 2t - 1 \simeq 2,2 \Rightarrow 2t \simeq 4,3 \Rightarrow t \simeq 2,2$

12a) $2^{5t+1} = 1024 \Rightarrow 5t + 1 = 10 \Rightarrow 5t = 9 \Rightarrow$
$\Rightarrow t = \frac{9}{5} = 1,8$
12b) $3 \cdot 2^t = 21 \Rightarrow 2^t = 7 \Rightarrow t \simeq 2,8$

t	2	3	2,5	2,6	2,7	2,8
2^t	4	8	5,7	6,1	6,5	7

12c) $2^{4-t} = 7 \Rightarrow 4 - t \simeq 2,8 \Rightarrow t \simeq 1,2$
12d) $\left(\frac{1}{3}\right)^{t-1} + 1 = 10 \Rightarrow$

$\left(\frac{1}{3}\right)^{t-1} = 9 \xrightarrow{9=3^2=\left(\frac{1}{3}\right)^{-2}} t - 1 = -2 \Rightarrow t = -1$

13)

t	0		1		2		3		4		5
hoeveelheid	B	$\xrightarrow{*2}$	2B	$\xrightarrow{*2}$	4B	$\xrightarrow{*2}$	8B	$\xrightarrow{*2}$	16B	$\xrightarrow{*2}$	32B

13a)

$\xrightarrow{4\ keer\ zo\ groot\ na\ 2\ uur} 2^2 = 4$

$\xrightarrow{8\ keer\ zo\ groot\ na\ 3\ uur} 2^3 = 8$

13b)

$\xrightarrow{32\ keer\ zo\ groot\ na\ 5\ uur} 2^5 = 32$

$$\boxed{2^p * 2^q = 2^{p+q}\ bijvoorbeeld\ 2^2 * 2^3 = 2^{2+3} = 2^5}$$

14) $H = 10^t$, t in weken, H in Miljoenen

14a) $t = 0 \Rightarrow H = 10^0 = 1$

$10^{t_1} = 2 \Rightarrow t_1 = {}^{10}\log 2 \simeq 0,3$

t	0,2	0,3	0,4
H	1,58	1,995	2,51

14b) $10^{t_2} = 10^1 \Rightarrow t_2 = 1$

14c) $10^t = 5 \Rightarrow t = {}^{10}\log 5$

$10^{t_1} \cdot 10^t = 10^{t_2} \Rightarrow 10^t = \frac{10^{t_2}}{10^{t_1}} = 10^{t_2 - t_1}$

14d) $t = t_2 - t_1 \Rightarrow t = 1 - 0,3 = 0,7$

Logaritmische Functies
Kern 2
Eigenschappen

15) $B = 100 \cdot 5^t$
15a) 2 keer zo groot$\longrightarrow 200 = 100 \cdot 5^{t_1} \;\Rightarrow\; t_1 = {}^5\log 2$
15b)3 keer zo groot$\longrightarrow 3 = 5^{t_2} \;\Rightarrow\; t_2 = {}^5\log 3$
15c) 6 keer zo groot$\longrightarrow t_1 + t_2 = {}^5\log 2 + {}^5\log 3 = {}^5\log 2 \cdot 3 = {}^5\log 6$
16a) ${}^3\log 2 + {}^3\log 4,5 \overset{E_{191}}{=} {}^3\log 2 \cdot 4,5 = {}^3\log 9 = 2$
16b) ${}^3\log 12 - {}^3\log 4 \overset{E_{192}}{=} {}^3\log \frac{12}{4} = {}^3\log 3 = 1$
16c) ${}^{10}\log 0,8 + {}^{10}\log 125 = {}^{10}\log 0,8 \cdot 125 = {}^{10}\log 1 = 0 \xrightarrow{Want} 10^0 = 1$
16d) ${}^5\log 4 - {}^5\log 100 = {}^5\log \frac{4}{100} = {}^5\log \frac{1}{25} = -2 \xrightarrow{Want} 5^{-2} = \frac{1}{25}$
17a) ${}^2\log 15 = {}^2\log 5 \cdot 3 = {}^2\log 5 + {}^2\log 3 \simeq 3,90$
17b) ${}^2\log \frac{5}{3} = {}^2\log 5 - {}^2\log 3 \simeq 0,74$
17c) ${}^2\log \frac{3}{5} = {}^2\log 3 - {}^2\log 5 \simeq -0,74$
17d) ${}^2\log \frac{1}{5} = {}^2\log 1 - {}^2\log 5 \simeq 0 - 2,32 \simeq -2,32$
17e) ${}^2\log \frac{1}{3} = {}^2\log 1 - {}^2\log 3 \simeq 0 - 1,58 \simeq -1,58$
17f) ${}^2\log \frac{1}{15} = {}^2\log 1 - {}^2\log 15 = 0 - {}^2\log 5 \cdot 3 = 0 - {}^2\log 5 - {}^2\log 3 \simeq 3,9$
17g) ${}^2\log 45 = {}^2\log 3 \cdot 3 \cdot 5 = {}^2\log 3 \cdot 3 + {}^2\log 5 = {}^2\log 3 + {}^2\log 3 + {}^2\log 5 \simeq 5,48$

18a)
${}^{10}\log 4 = {}^{10}\log 2 \cdot 2 = {}^{10}\log 2 + {}^{10}\log 2 \simeq 0,602$
${}^{10}\log 5 = {}^{10}\log 10 \cdot \frac{1}{2} = {}^{10}\log 10 + {}^{10}\log \frac{1}{2} = 1 + {}^{10}\log 1 - {}^{10}\log 2 \simeq 1 + 0 - 0,301 \simeq 0,699$
${}^{10}\log 6 = {}^{10}\log 2 \cdot 3 = {}^{10}\log 2 + {}^{10}\log 3 \simeq 0,778$
${}^{10}\log 8 = {}^{10}\log 2 \cdot 2 \cdot 2 = {}^{10}\log 2 + {}^{10}\log 2 + {}^{10}\log 2 = 3 \cdot {}^{10}\log 2 \simeq 0,903$
${}^{10}\log 9 = {}^{10}\log 3 \cdot 3 = {}^{10}\log 3 + {}^{10}\log 3 \simeq 0,4771 + 0,4771 \simeq 0,954$
18b) ${}^{10}\log 1 \quad {}^{10}\log 12 \quad {}^{10}\log 15 \quad {}^{10}\log 16 \quad {}^{10}\log 18$

19a) ${}^3\log 5 + 2 = {}^3\log 5 + {}^3\log 9 = {}^3\log 45$
19b) $1 + {}^5\log 2 = {}^5\log 5 + {}^5\log 2 = {}^5\log 10$
19c) $3 \cdot {}^2\log 5 + 3 = 3 \cdot \left({}^2\log 5 + 1\right) = 3 \cdot \left({}^2\log 5 + {}^2\log 2\right) = 3 \cdot {}^2\log 5 \cdot 2 = 3 \cdot {}^2\log 10$
19d) ${}^{12}\log 120 - {}^{12}\log 12 = {}^{12}\log \frac{120}{12} = {}^{12}\log 10$

20a) $4 \cdot {}^2\log 5 = {}^2\log 5 + {}^2\log 5 + {}^2\log 5 + {}^2\log 5 = {}^2\log 25 + {}^2\log 25 = {}^2\log 25 \cdot 25 = {}^2\log 5^4 = 625$
20b) $3 \cdot {}^5\log 4 = {}^5\log 4 + {}^5\log 4 + {}^5\log 4 = {}^5\log 4 \cdot 4 \cdot 4 = {}^5\log 4^3 = {}^5\log 64$

21a) $2 \cdot {}^3\log 5 = {}^3\log 5^2 = {}^3\log 25$
21b) $-5 \cdot {}^7\log 2 = {}^7\log 2^{-5} = {}^7\log \frac{1}{2^5} = {}^7\log \frac{1}{32}$
21c) $3 \cdot {}^2\log \frac{1}{3} = {}^2\log \left(\frac{1}{3}\right)^3 = {}^2\log \frac{1}{27}$
21d) $2 \cdot {}^{0,5}\log 25 = {}^{0,5}\log 25^2 = {}^{0,5}\log 625$
21e) $3 \cdot {}^{\frac{1}{8}}\log 2 = {}^{\frac{1}{8}}\log 2^3 = {}^{\frac{1}{8}}\log 8 = {}^{\frac{1}{8}}\log \left(\frac{1}{8}\right)^{-1} = -1$
21f) $2 \cdot {}^{10}\log 2 + {}^{10}\log 6 = {}^{10}\log 2^2 + {}^{10}\log 6 = {}^{10}\log 4 = {}^{10}\log 2 \cdot 6 = {}^{10}\log 24$
21g) ${}^5\log 24 - 3 \cdot {}^5\log 2 = {}^5\log 24 - {}^5\log 2^3 = {}^5\log 24 - {}^5\log 8 = {}^5\log \frac{24}{8} = {}^5\log 3$
21h) $2 \cdot {}^4\log 100 - {}^4\log 200 = {}^4\log 100^2 - {}^4\log 200 = {}^4\log 10.000 - {}^4\log 200 = {}^4\log \frac{10.000}{200} = {}^4\log 50$

22a) $\log 1000 \equiv^{10} \log 1000 = 3$

22b) $10 \xrightarrow{Want} 10^3 = 1000$

22c) $\log \sqrt{10} = \frac{1}{2} \log 0,1 = -1$

23a) ${}^2\log 7 \simeq 2,8074 \xrightarrow{exact} \left(\frac{\log 7}{\log 2}\right)$

23b) ${}^2\log 70 \simeq 6,1293 \xrightarrow{exact} \left(\frac{\log 70}{\log 2}\right)$

23a) ${}^2\log 700 \simeq 9,4512 \xrightarrow{exact} \left(\frac{\log 700}{\log 2}\right)$

23a) ${}^2\log 5 \simeq 2,3219 \xrightarrow{exact} \left(\frac{\log 5}{\log 2}\right)$

23a) ${}^{0,2}\log 7 \simeq -1.2091 \xrightarrow{exact} \left(\frac{\log 7}{\log 0,2}\right)$

23a) ${}^7\log 0,2 \simeq -0,8271 \xrightarrow{exact} \left(\frac{\log 0,2}{\log 7}\right)$

24a)

5000	$\xrightarrow{*1,1}$	5500
100%	$\xrightarrow{*1,1}$	110%

$\xrightarrow{Groeifactor} 1,1$

24b)

$1\,jaar \longrightarrow \quad 5000 \cdot 1,1 \quad = Hfl\,5500,-$

$2\,jaar \longrightarrow \quad 5000 \cdot 11 \cdot 1,1 \quad = 5000 \cdot 1,1^2 \quad = Hfl\,6050,-$

$20\,jaar \longrightarrow \quad 5000 \cdot 1,1^{20} \quad \simeq Hfl\,33.637,50$

24c) $5000 \cdot (1,1)^t = 9000 \;\Rightarrow\; (1,1)^t = \frac{9000}{5000} = 1,8 \;\Rightarrow\; t = {}^{1,1}\log 1,8 \;\Rightarrow\; t = \frac{\log 1,8}{\log 1,1} \simeq 6,167$

24d)

$$\left.\begin{array}{lll} t=0 & 1\,mei & 1998 \\ t=6 & 1\,mei & 2004 \\ t=6,167 & 1\,juli & 2004 \end{array}\right\} \longrightarrow 0,167 \cdot 12 \simeq 2\,maand$$

25) Toename 5% per jaar$\xrightarrow{Groeifactor} 1,05$

100%	$\xrightarrow{*1,05}$	105%

$\xrightarrow{Groeifactor} 1,05$

$t = 0 \xrightarrow{1\text{ januari }1996} 1000\,dieren \;\Rightarrow\; D = 1000 \cdot 1,05^t \xrightarrow{t\text{ in jaren}} 2000 = 1000 \cdot 1,05^t \;\Rightarrow\; 1,05^t = 2$

$\Rightarrow\; t = {}^{1,05}\log 2 = \frac{\log 2}{\log 1,05} \simeq 14,21 \xrightarrow{Dus} 1996 + 14 = 2010$

In het jaar 2010 zijn er voor het eerst meer dan 2000 dieren.

26) $A(t) = 34,3 \cdot (1,02)^t$ A in miljoenen, t in jaren, $t = 0 \longrightarrow$ 1 januari 1995

$50 = 34,3 \cdot (1,02)^t \;\Rightarrow\; (1,02)^t = \frac{50}{34,3} \;\Rightarrow\; t = {}^{1,02}\log\left(\frac{50}{34,3}\right) \;\Rightarrow$

$\Rightarrow\; t = \frac{\log\left(\frac{50}{34,3}\right)}{\log 1,02} \simeq \frac{\log 1,458}{\log 1,02} \simeq 19 \xrightarrow{Dus} 1995 + 19 = 2014$

27) Groeifactor per kwartier is 3; $t = 0\,bij\,10m^2$; t in kwartieren

$300 = 10 \cdot 3^t \;\Rightarrow\; 3^t = 30 \;\Rightarrow\; t = {}^3\log 30 = \frac{\log 30}{\log 3} \simeq 3,1 \xrightarrow{Dus} 3\,kwartier + 0,1 \cdot 15 \simeq 46,5\,minuten$

$8.15uur - 0.465 \simeq 7.29uur$

28a) $2^t = 3 \xrightarrow{t\text{ in uren}} t = {}^3\log 3 = \frac{\log 3}{\log 2} \simeq 1,58 \;\Rightarrow\; 12 + 0,58 \cdot 12 \simeq 19\,uur$

28b) $2^{t_1} = 9 \xrightarrow{9=3*3} 2^{t_1} = 3 \cdot 3 \;\Rightarrow\; 2^{t_1} = 2^t \cdot 2^t \;\Rightarrow\; 2^{t_1} = 2^{t+t} \;\Rightarrow\; t_1 = 19 + 19 = 38\,uur$

28c) $50\,uur = 19 + 19 + 12$

$2^{19+19+12} = 2^{19} \cdot 2^{19} \cdot 2^{12} = 3 \cdot 3 \cdot 2 = 18\,keer\,zo\,groot$

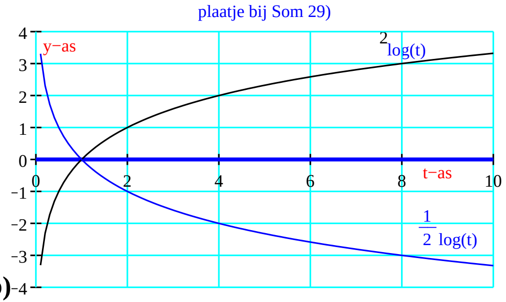
Logaritmische Functies

Kern 3
Grafieken

29a)

t	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8
${}^{2}\log t$	-3	-2	-1	0	1	2	3
${}^{\frac{1}{2}}\log t$	3	2	1	0	-1	-2	-3

29b)



29c) $D_f = \langle 0, \rightarrow \rangle$

29d) $y = 0$

30a)

$$\left.\begin{array}{r} f(x) = {}^{g}\log x \\ 1^{ste} grafiek : (3; -1) \end{array}\right\} \Rightarrow -1 = {}^{g}\log 3 \Rightarrow g^{-1} = 3 \Rightarrow \frac{1}{g} = 3 \Rightarrow g = \frac{1}{3}$$

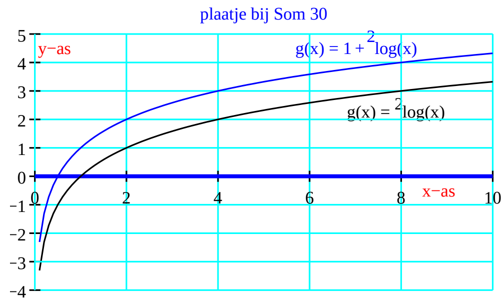
$$\left.\begin{array}{r} f(x) = {}^{g}\log x \\ 2^{de} grafiek : (3; 1) \end{array}\right\} \Rightarrow 1 = {}^{g}\log 3 \Rightarrow g^{1} = 3$$

$$\boxed{{}^{g}\log a = b \Rightarrow g^{b} = a}$$

30b) ${}^{\frac{1}{2}}\log x = \frac{\log x}{\log \frac{1}{2}} = \frac{\log x}{\log 2^{-1}} \xrightarrow{Zie\,Eigenschap\,3\,(bldz\,88)} {}^{\frac{1}{2}}\log x = \frac{\log x}{-\log 2} = -\frac{\log x}{\log 2} = -{}^{2}\log x$

31)
$f(x) = {}^{2}\log x$
$g(x) = 1 + {}^{2}\log x$

x	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8
${}^{2}\log x$	-2	-1	0	1	2	3
$1 + {}^{2}\log x$	-1	0	1	2	3	4



31b) "1 omhoog transleren"

31c) $D_g : \langle 0; \rightarrow \rangle$ en $B_g = \Re$

31d) $\xrightarrow{Asymptoot} x = 0$

32a)

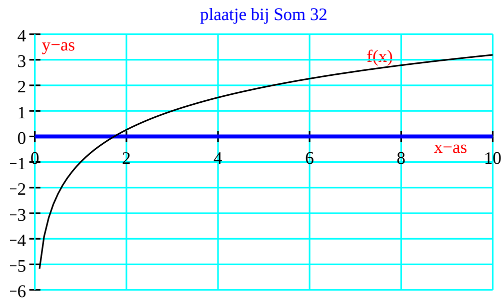
$$\left.\begin{array}{r} f(x) = b + a \cdot {}^{3}\log x \\ A : (1; -1) \end{array}\right\} \Rightarrow -1 = b + a \cdot {}^{3}\log 1$$

$\Rightarrow -1 = b + a \cdot 0 \Rightarrow b = -1$

$$\left.\begin{array}{r} f(x) = -1 + a \cdot {}^{3}\log x \\ B : (3; 1) \end{array}\right\} \Rightarrow 1 = -1 + a \cdot {}^{3}\log 3$$

$\Rightarrow 2 = a \cdot 1 \Rightarrow a = 2$

$f(x) = -1 + 2 \cdot {}^{3}\log x$



32c)

x	$\frac{1}{9}$	$\frac{1}{3}$	1	3	9	27
	-5	-3	-1	1	3	5

33)

x	$-\frac{3}{4}$	$-\frac{1}{2}$	0	$\frac{1}{4}$	$\frac{1}{2}$	1	2	3	4
${}^{2}\log x$	–	–	–	-2	0	0	1	1,58	2
${}^{2}\log(x+1)$	-2	-1	0	0,32	0,58	1	1,58	2	2,32

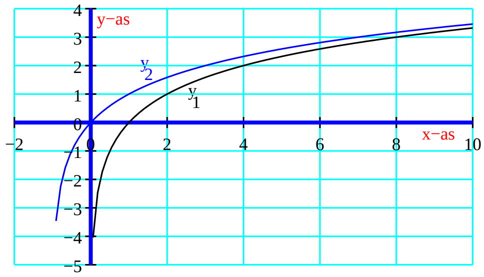
33c) Vorm is gelijk $^{2}\log(x+1)$ is 1 verschoven naar links

33d)

$y_1 : D = \langle 0; \rightarrow\rangle \; B = \Re \; Asymptoot : x = 0$

$y_2 : D = \langle -1; \rightarrow\rangle \; B = \Re \; Asymptoot : x = -1$

plaatje bij Som 33



34a) $f(x) = ^{2}\log(x-3)$

$x - 3 > 0 \Rightarrow x > 3 \; D_f : \langle 3; \rightarrow\rangle$

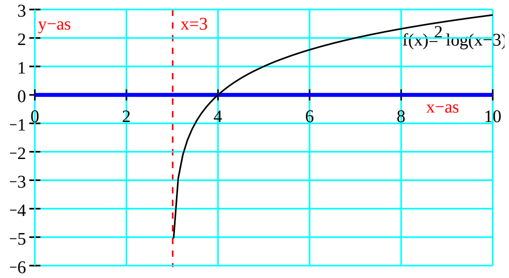
$Asymptoot : x = 3$

$\xrightarrow{Snijpunt\ met\ x-as} {}^{2}\log(x-3) = 0 \Rightarrow$

$\Rightarrow x - 3 = 1 \Rightarrow x = 4 \xrightarrow{Het\ Punt} (4;0)$

x	5	4
$f(x)$	1	2

plaatje bij Som 34a)



34b) $g(x) = ^{2}\log(4-x) + 2$

$4 - x > 0 \Rightarrow x < 4 \; D_g : \langle \leftarrow; 4\rangle$

$Asymptoot : x = 4$

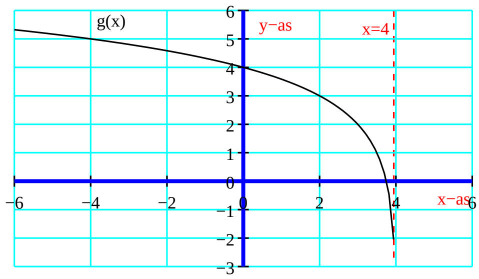
$\xrightarrow{Snijpunt\ met\ x-as} {}^{2}\log(4-x) + 2 = 0 \Rightarrow$

${}^{2}\log(4-x) = -2 \Rightarrow 4 - x = \frac{1}{4} \Rightarrow$

$\Rightarrow x = 3\frac{3}{4} \xrightarrow{Het\ Punt} (3\frac{3}{4};0)$

x	-4	0	2	3
$g(x)$	5	4	3	2

plaatje bij Som 34b)



34c) $h(x) = -1 + ^{\frac{1}{2}}\log x$

$D_h : \langle 0; \rightarrow\rangle$

$Asymptoot : y = 0 \; x - as$

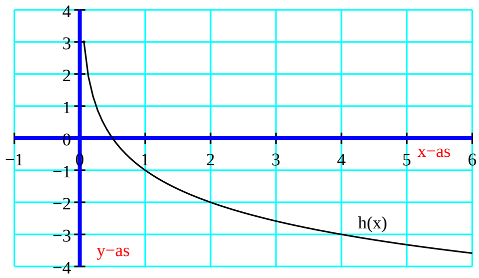
$\xrightarrow{Snijpunt\ met\ x-as} {}^{\frac{1}{2}}\log(x) - 1 = 0 \Rightarrow$

${}^{\frac{1}{2}}\log(x) = 1 \Rightarrow x = \frac{1}{2} \Rightarrow$

$\xrightarrow{Het\ Punt} (\frac{1}{2};0)$

x	$\frac{1}{4}$	1	2	4
$h(x)$	1	-1	-2	-3

plaatje bij Som 34c)



34d) $k(x) = 2 - ^{\frac{1}{2}}\log(2x)$

$2x > 0 \Rightarrow x > 0 \; D_k : \langle 0; \rightarrow\rangle$

$Asymptoot : x = 0$

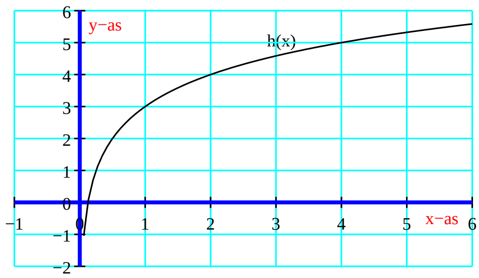
$\xrightarrow{Snijpunt\ met\ x-as} 2 - ^{\frac{1}{2}}\log(2x) = 0 \Rightarrow$

${}^{\frac{1}{2}}\log(2x) = 2 \Rightarrow 2x = \frac{1}{4} \Rightarrow$

$\Rightarrow x = \frac{1}{8} \xrightarrow{Het\ Punt} (\frac{1}{8};0)$

x	$\frac{1}{4}$	$\frac{1}{2}$	1	2
$k(x)$	1	2	3	4

plaatje bij Som 34d)



35a) $f(x) = {}^3\log(x-4)$

$x-4>0 \;\Rightarrow\; x>4 \; D_f : \langle 4;\rightarrow\rangle$

$Asymptoot : x = 4$

$\xrightarrow{Snijpunt\ met\ x-as} {}^3\log(x-4) = 0 \;\Rightarrow$

$x-4=1 \;\Rightarrow\; x=5 \xrightarrow{Het\ Punt} (5;0)$

$g(x) = {}^3\log(4-x)$

$4-x>0 \;\Rightarrow\; x<4 \; D_g : \langle \leftarrow;4\rangle$

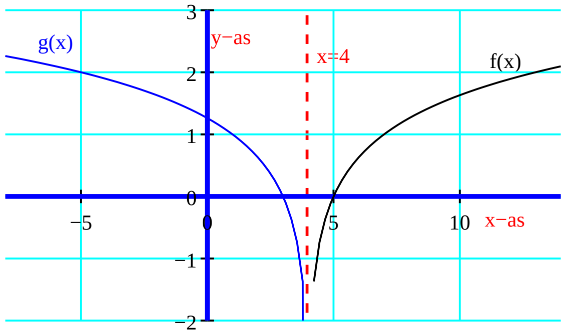
$Asymptoot : x = 4$

$\xrightarrow{Snijpunt\ met\ x-as} {}^3\log(4-x) = 0 \;\Rightarrow$

$4-x=1 \;\Rightarrow\; x=3 \xrightarrow{Het\ Punt} (3;0)$

35b) Spiegeling in de asymptoot: $x = 4$

plaatje bij Som 35)



36a) $f(x) = {}^2\log(2x)+1$

$2x>0 \;\Rightarrow\; x>0 \; D_f : \langle 0;\rightarrow\rangle$

$Asymptoot : x = 0$

$\xrightarrow{Snijpunt\ met\ x-as} {}^2\log(2x)+1 = 0 \;\Rightarrow$

$2x = \frac{1}{2} \;\Rightarrow\; x = \frac{1}{4} \xrightarrow{Het\ Punt} (\frac{1}{4};0)$

$g(x) = {}^2\log(5-x)$

$5-x>0 \;\Rightarrow\; x<5 \; D_g : \langle \leftarrow;5\rangle$

$Asymptoot : x = 5$

$\xrightarrow{Snijpunt\ met\ x-as} {}^3\log(5-x) = 0 \;\Rightarrow$

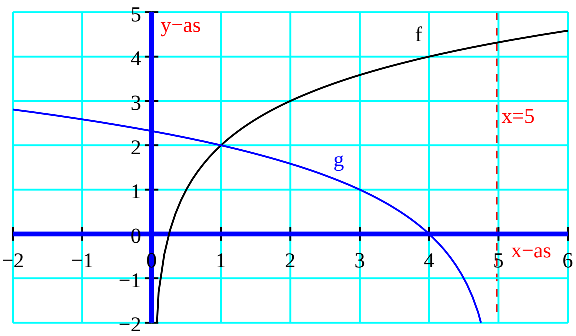
$5-x=1 \;\Rightarrow\; x=4 \xrightarrow{Het\ Punt} (4;0)$

x	1	2	4
$f(x)$	2	3	3,6
$g(x)$	2	1,6	1

36b) $f(x) = g(x) \;\Rightarrow\; x = 1 \xrightarrow{het\ Punt} (1;2)$

36c) $f(x) > g(x) \;\Rightarrow\; x \in \langle 1;5\rangle$

plaatje bij Som 36)



37) $G = -3 + 10\cdot\log(200\cdot t)$

$G = -3 + 10\cdot\log(200\cdot t) \;\Rightarrow$

$G = -3 + 10\cdot(\log 200 + \log t) \;\Rightarrow$

$G = -3 + 10\cdot\log 2\cdot 100 + 10\cdot\log t \;\Rightarrow$

37a) $G = -3 + 10\log 2 + 10\log 100 + 10\log t \;\Rightarrow$

$G = -3 + 10\log 2 + 20 + 10\log t \;\Rightarrow$

$G = 17 + 10\log 2 + 10\log t \;\Rightarrow$

$G \simeq 20 + 10\log t$

37b) $5,0\,sec$

37c) $t = 0,01\ sec$

$G = 20 + 10\log 0,01 = 0 \xrightarrow{10\log 0,01 = \log 10^{-2} = -2} Klopt$

38a) $m = 5 \cdot \log R - 2,8$ waarin R is de afstand tot de aarde in lichtjaar

R	$\frac{1}{10}$	1	2	3	4	5	6	7	8	9	10
m	-7,8	-2,8	-1,3	0,4	0,2	0,7	1,1	1,4	1,7	1,97	2,2

38b) Een ★ , die ver weg staat (R groot) heeft ook een grote m-waarde, betekent een kleine helderheid.

38c) $0 = 5 \cdot \log R - 2,8 \Rightarrow 5 \cdot \log R = 2,8 \Rightarrow \log R = \frac{2,8}{5} = 0,56$

$R = 10^{0,56} \simeq 3,6$

$3,6\, licht\, jaar \simeq 34,4 \times 10^{12} kilometer$

38d)

$$\left.\begin{array}{r} \bigstar A \longrightarrow m_A = 5 \cdot \log 2 \cdot R - 2,8 \Rightarrow 5 \cdot (\log 2 + \log R) - 2,8 \Rightarrow 5\log 2 + 5\log R - 2,8 \\ \bigstar B \longrightarrow m_B = 5\log R - 2,8 \end{array}\right\} \Rightarrow$$

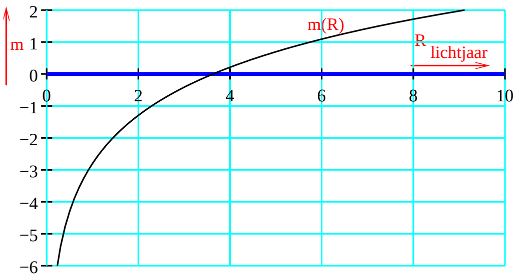
$\Rightarrow m_A - m_B = 5\log 2 \simeq 1,51$

38e) $(R;m)$ $(5;0,7)$ $(8;1,7)$ $\frac{8}{5} = 1,6$

38f) $m_p = m_q + 1$

$m_p = 5\log R - 2,8 + 1 \Rightarrow m_p = 5\left(\log R + \log 10^{\frac{1}{5}}\right) - 2,8 \Rightarrow m_p = 5(\log 1,58R) - 2,8$

plaatje bij Somn 38)



KERN 4
LOGARITMISCH PAPIER

39a) Onderlinge verschillen zijn te groot
0,01mm en 10km zijn niet te zien bij een grote schaal; bij een kleine schaal komen 1000km en 100.000km ver buiten het papier.

39b)

PC							stad				A		Z	
10^{-8}	10^{-7}	10^{-6}	10^{-5}	10^{-4}	10^{-3}	10^{-2}	10^{-1}	10^{1}	10^{2}	10^{3}	10^{4}	10^{5}	10^{6}	km

$10^{-8} kilometer = 0,000.000.01 kilometer \overset{*1000}{=} 0,000.01 meter \overset{*100}{=} 0,001 cm \overset{*10}{=} 0,01 mm$

39c)

PC							stad				A		Z
10^{-8}	10^{-7}	10^{-6}	10^{-5}	10^{-4}	10^{-3}	10^{-2}	10^{-1}	10^{1}	10^{2}	10^{3}	10^{4}	10^{5}	10^{6}
-8	-7	-6	-5	-4	-3	-2	-1	1	2	3	4	5	6

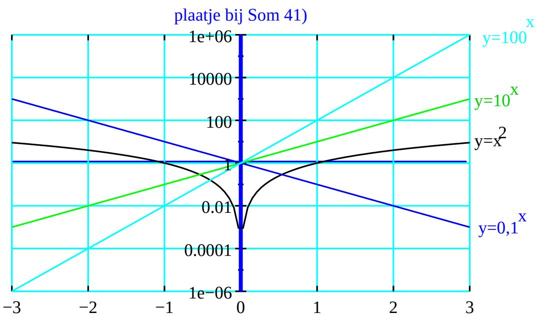
40)

-4		-3		-2		-1		0		1		2		3		4
10^{-4}		10^{-3}		10^{-2}		10^{-1}		10^{0}		10^{1}		10^{2}		10^{3}		10^{4}
	0,0004↑					0,1	0,2		$\sqrt{10}$				500↑	1000	8000↑	

40a) $1000 \longrightarrow^{10} \log 1000 = 3$
40b) $500 \longrightarrow^{10} \log 500 \simeq 2,7$
40c) $0,1 \longrightarrow \log 0,1 = -1$
40d) $0,2 \longrightarrow \log 0,2 \simeq -0,7$

40e) $\log 8000 \simeq 3,9$
40f) $\log 45000 \simeq 4,7$
40g) $\log \sqrt{10} = 0,5$
40h) $\log 0,0004 \simeq -3,4$

41a)
p ligt precies tussen 10^1 *en* 10^2 in
$p = 10^{1,5} \simeq 31,6$



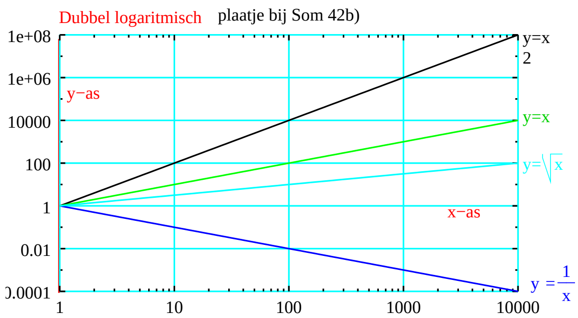
42a)

$q = 10^{2,5} \simeq 316,2$

$x = 10^{3,25} \simeq 1778,2$

$y = 10^{3,75} \simeq 5623,4$

42b)



x	$\frac{1}{4}$	$\frac{1}{2}$	1	2	3	4
$y = x^2$	$\frac{1}{16}$	$\frac{1}{4}$	1	4	9	16
$\log x^2$	-1,2	-0,6	0	0,6	0,95	1,2

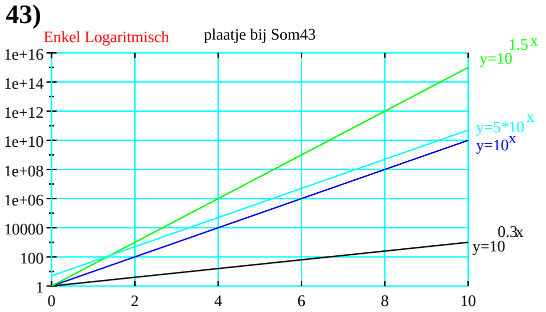
42b) vervolg

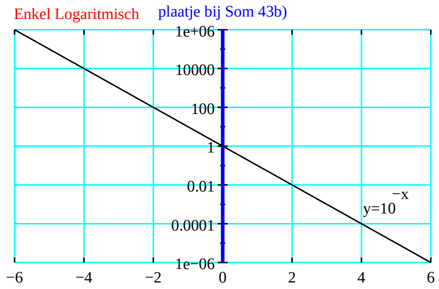
x	1	5	10
$\log x$	0	0,7	1
x^2	1	25	100
$\log x^2$	0	1,4	2

x	1	10	100
$\log x$	0	1	2
$\frac{1}{x}$	1	$\frac{1}{10}$	$\frac{1}{100}$
$\log \frac{1}{x}$	0	-1	-2

x	1	100	10.000
$\log x$	0	2	4
$\sqrt{x}$	1	10	100
$\log \sqrt{x}$	0	1	2

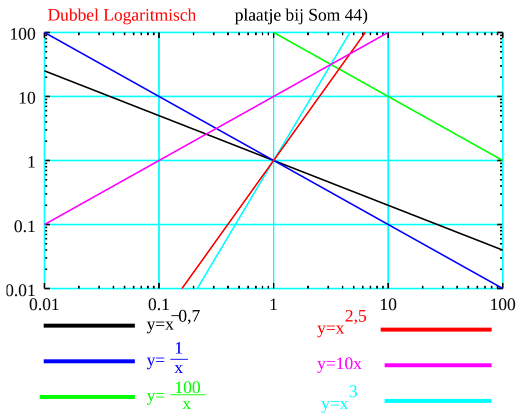
43)





44) Tabel

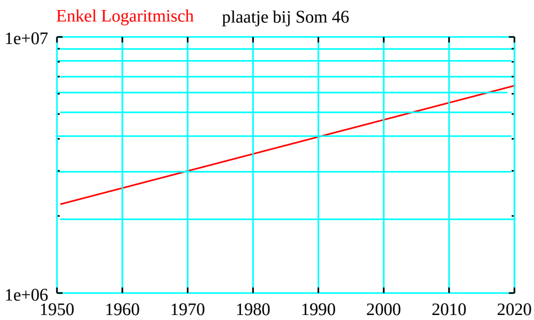
x	0,1	1	2	3	4	6	10	100
$10x$	1	10					$100=10^2$	
x^3		1	8	27	64			
$x^{2,5}$		1	5,66		32	88,2	316,2	$100.000=10^5$
$\frac{1}{x}$	10	1					$\frac{1}{10}=10^{-1}$	$\frac{1}{100}=10^{-2}$
$10^{-0,7}$	5,0	1					0,2	0,04
$\frac{100}{x}$	1000	100					10	1



45a) 5000 *à* 600 *jaar*
45b) 12000 *jaar*

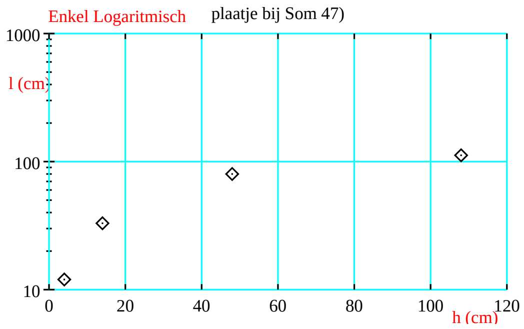
46a)

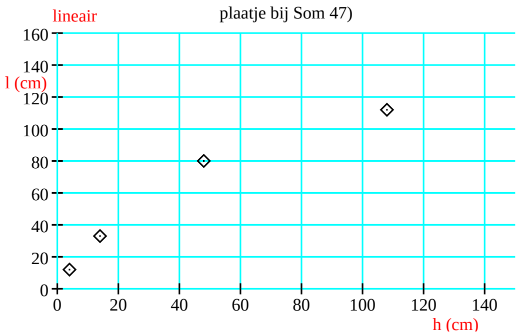
1960	30×10^6	inwoners
1990	50×10^6	inwoners

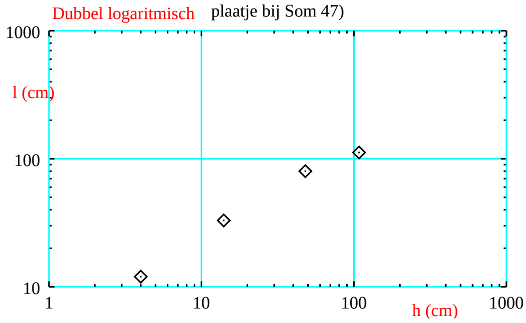


46b) $\simeq 70\,miljoen$

47a) Geen lineair verband
47b) Geen exponentieel verband
47c) Rechte lijn$\longrightarrow$ *dus* een machtsfunctie







Grafische Rekenmachine

1a) $f(x)=\log x$

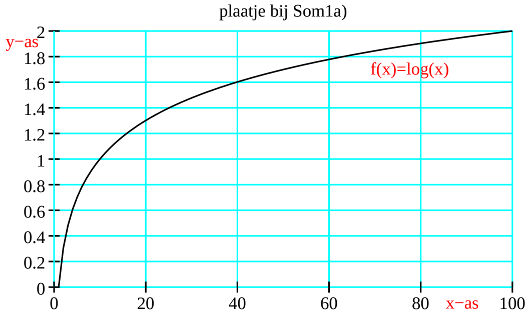
1b) $f(x)=0,7 \xrightarrow{0,7=\log x} x=10^{0,7}\simeq 5,0$

1c) $f(x)<0,6 \Rightarrow x\in\langle 0;5,01\rangle$

g(x) < < < < < = > > > > > > 0,7

5.0

plaatje bij Som1a)



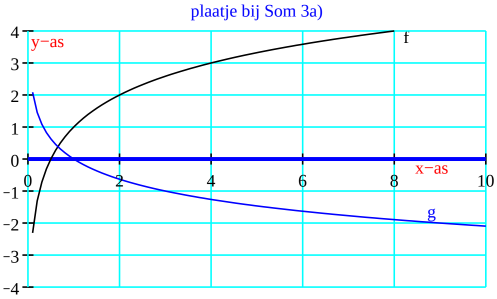
2a) $f(x)={}^3\log x=\frac{\log x}{\log 3}=\frac{y_1}{\log 3}$
2b) ${}^3\log x=1,5 \Rightarrow x=3^{1,5}\simeq 5,2$
$f(x)<1,5 \Rightarrow x\in\langle 0;5,2\rangle$

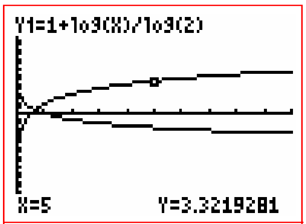
^{3} g(x) < < < < < < < = > > > > > > 0,7

5.2

3a) $f(x)=1+{}^2\log x\ \ g(x)={}^{\frac{1}{3}}\log x$
3b) $(0,65:0,39)$
4) $f(x)=1,2-{}^3\log x$

plaatje bij Som 3a)





Screendump Ti83

plaatje bij Som 3a)

DOORWERKING

D1) $\log A = 2,15 - 0,13n \; n \geqq 2$
D1a) $n = 2 \Rightarrow \log A = 2,15 - 0,13 \cdot 2 = 1,89 \Rightarrow A = 10^{1,89} \simeq 77,6$
D1b)
$1^{ste} dag \longrightarrow 200$
$2^{de} dag \longrightarrow 200 + 78 = 278$
$3^{de} dag \longrightarrow 200 + 78 + 58 = 336$
$n = 3 \Rightarrow \log A = 2,15 - 0,13 \cdot 3 = 1,76 \Rightarrow A = 10^{1,76} \simeq 57,5$
Dus 58 producten meer
$1^{ste} drie\ dagen \longrightarrow 200 + 278 + 336 = 814$
D1c) Aan het begin van de inwerkperiode
want dan is de meer_productie per dan hoger
D1d)

$$\left.\begin{matrix} \log A = 2,15 - 0,13 \cdot n \\ A = p \cdot g^n \end{matrix}\right\} \Rightarrow log\,(p \cdot g)^n = 2,15 - 0,13n \Rightarrow$$

$$\Rightarrow \log p + \log g^n \overset{Eigenschap_1}{=} 2,15 - 0,13n \Rightarrow$$

$$\Rightarrow \log p + n \log g \overset{Eigenschap_3}{=} 2,15 - 0,13n \xrightarrow{Stel\ n=0} \log p = 2,15 \Rightarrow p = 10^{2,15} \simeq 141$$

$$\xrightarrow{Als\ n=1} \log p + \log g = 2,15 - 0,13 \xrightarrow{p=10^{2,15}} 2,15 + \log g = 2,15 - 0,13 \Rightarrow$$

$$\Rightarrow \log g = -0,13 \Rightarrow g = 10^{-0,13} \simeq 0,74$$

D2) $N = 10 \cdot \log I + 120$
D2a) Rustige dag$\longrightarrow I = 10^{-7} W/m^2 \Rightarrow N = 10 \log 10^{-7} + 120 \Rightarrow N = -70 + 120 = 50dB$
D2b) $50 + 30 = 80dB$
$80 = 10 \log I + 120 \Rightarrow 10 \log I = -40 \Rightarrow \log I = -4 \Rightarrow I = 10^{-4} W/m^2$
D2c) $0 = 10 \log I + 120 \Rightarrow 10 \log I = -120 \Rightarrow \log I = -12 \Rightarrow I = 10^{-12} W/m^2$
D2d) Verdubbeling van de Geluidsintensiteit: $I \longrightarrow 2 \cdot I$

$$N \overset{Eigenschap_1}{=} 10\,(\log 2 + \log I) + 120 \Rightarrow N = 10 \log 2 + 10 \log I + 120 \simeq 3 + 10 \log I + 120$$

Bij verdubbeling van I meent N dus met 3 toe.
D2e) $80 = 10 \log I + 120 \Rightarrow -40 = 10 \log I \Rightarrow -4 = \log I \Rightarrow I = 10^{-4} \Rightarrow$
$\Rightarrow N = 10 \cdot \log 16 \cdot 10^{-4} + 120 \simeq 92dB$

D3a) Per kilogram$\longrightarrow 20\,milligram \Rightarrow 20kg \longrightarrow 20 * 20 = 400milligram$
Groeifactor per uur:

$$\tfrac{1}{2} \longrightarrow N = B\left(\tfrac{1}{2}\right)^t \xrightarrow{waarbij\ t\ per\ 2\ uur} 400 = B\left(\tfrac{1}{2}\right)^{\frac{70}{120}} \Rightarrow B = \frac{400}{\left(\frac{1}{2}\right)^{\frac{70}{120}}} \simeq 599mg$$

D3b) Het is moeilijk in te schatten hoe lang de operatie zal duren.
D3c) $N = B\left(\frac{1}{2}\right)^t$

t	0	30		30	60	90	120
N	500	420,4	$\overrightarrow{100\,mg\,erbij}$	520,4	437,6	368	309,5

D3d) $N = 500 \cdot \left(\frac{1}{2}\right)^{\frac{t}{120}}$

$waarbij; t \geqq 0,\ t\ in\ minuten$

D3e) $N_2 = 520,4 \cdot \left(\frac{1}{2}\right)^{\frac{t-30}{120}}$

D3f) Op $t = 70 \longrightarrow N \simeq 413mg\overleftarrow{Dus\,genoeg}$