

UITWERKINGEN VOOR HET VWO A1B1 DEEL1

Hoofdstuk 2

KERN 1 EXPONENTIEEL FUNCTIES

1a) $\left. \begin{array}{l} \text{Opp.Cirkel} = \pi r^2 \\ \text{Diameter} = 6 \Rightarrow r = 3 \end{array} \right\} \Rightarrow O_{\text{oppervlakte}} = 9\pi \simeq 28,27m^2$

1b) Toename per jaar 10%

t	0	1	2	3	4	5	6	7	8	9	10
O	9π	31,10	34,21	37,63	42,40	45,54	50,09	55,10	60,61	66,67	73,34

1c) $g = 1,1$

2a)

t	0	1	2	3	4
H	2	6	18	54	162

2b)

t	0	1	2	3	4
H	12	3	$\frac{3}{4}$	$\frac{3}{16}$	$\frac{3}{64}$

2e)

t	0	1	2	3	4
H	18	6	2	$\frac{2}{3}$	$\frac{2}{9}$

$\searrow * \frac{1}{3} \nearrow$

2d)

t	0	1	2	3	4
H	8	12	18	27	40,5

$\searrow * 1\frac{1}{2} \nearrow$

3a)

t	0	1	2	3	4	5
K	200000	212000	224720	238203,2	252495,39	267645,12

3b) $g = 1,06$

3c) $200000 \cdot 1,06^{10} \simeq 358169,54$

3d) $1,06^{10} \simeq 1,79$

4)

groeipercentage	10	1	100	250	3	120	-5	-0,2
groEIFactor	1,1	1,01	$2_{\text{opm.}_1}$	$3,5_{\text{opm.}_2}$	1,03	2,2	$0,95_{\text{opm.}_3}$	$0,998_{\text{opm.}_4}$

$\text{opm.}_1 \rightarrow \text{Oud } 100\% \xrightarrow{\text{maal } 2} \text{Nieuw } 100\% + 100\% = 200\%$

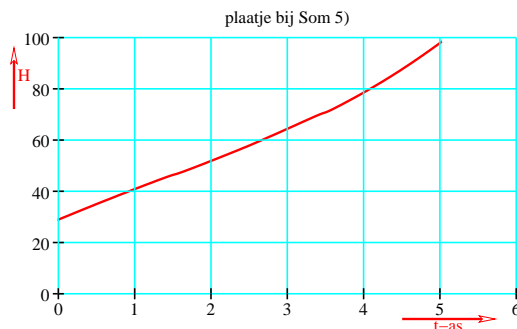
$\text{opm.}_2 \rightarrow \text{Oud } 100\% \xrightarrow{\text{maal } 3,5} \text{Nieuw } 100\% + 250\% = 350\%$

$\text{opm.}_3 \rightarrow \text{Oud } 100\% \xrightarrow{\text{maal } 0,95} \text{Nieuw } 95\% = 100\% - 5\%$

$\text{opm.}_4 \rightarrow \text{Oud } 100\% \xrightarrow{\text{maal } 0,998} \text{Nieuw } 99,8\% = 100\% - 0,2\%$

5)

t	0	1	2	3	4	5
H	32	40	50	62,5	78,125	97,66



¹ Deze samenvatting mag niet massaal op kosten van Schaersvoorde worden Uitgeprint!!!



² werd gemaakt onder Linux met \LaTeX en \LyX

³ Typ&andere fouten&blunders graag Melden!!

$$\begin{array}{l}
 \mathbf{6a)} H = 0,8 \cdot (1,25)^t \quad \left. \begin{array}{l} g = 1,25 \\ b = 0,8 \end{array} \right\} \xrightarrow{1,25 > 1} \text{dus Groei} \\
 \mathbf{6b)} H = 5,8 \cdot \left(\frac{1}{2}\right)^t \quad \left. \begin{array}{l} g = \frac{1}{2} \\ b = 5,8 \end{array} \right\} \xrightarrow{\frac{1}{2} < 1} \text{dus Verval} \\
 \mathbf{6c)} H = 1,03 \cdot (0,67)^t \quad \left. \begin{array}{l} g = 0,67 \\ b = 1,03 \end{array} \right\} \xrightarrow{0,67 < 1} \text{dus Verval} \\
 \mathbf{6d)} H = 1 \cdot (0,99)^t \quad \left. \begin{array}{l} g = 0,99 \\ b = 1 \end{array} \right\} \xrightarrow{0,99 < 1} \text{dus Verval}
 \end{array}$$

7a) $b = \text{Hfl } 500,-$

7b) $g = 1,05$

7c) $K = 500 \cdot (1,05)^t$

7d)

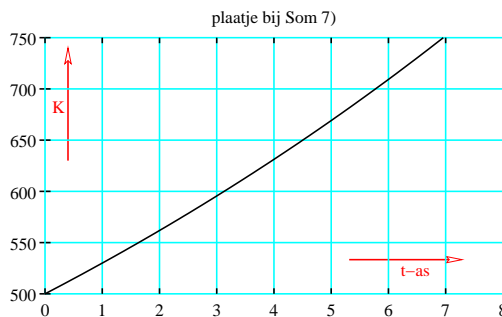
t	0	1	2	3	4	5	6	7	8
K	500	525	551,25	578,81	607,75	638,14	670,05	703,55	738,73

7e)

$$500 \cdot 1,05^9 \approx 775,66$$

$$500 \cdot 1,05^{10} \approx 814,45$$

Na 10 jaar



8a)

$$\frac{10476}{12325} \approx 0,85$$

$$\frac{8905}{10476} \approx 0,85$$

$$\frac{6434}{7569} \approx 0,85$$

8b) $b = 12325 \rightarrow B = 12325 \cdot 0,85^t$

8c) In 1990: $t = 0$

In 2100: $t - 110 \Rightarrow B = 12325 \cdot 0,85^{110} \Rightarrow B = 2 \cdot 10^{-4} \Rightarrow B = 0,0002 \xrightarrow{\text{Dus...}} 0 \text{ mensen}$

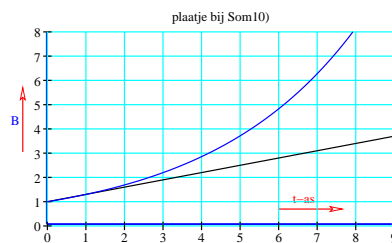
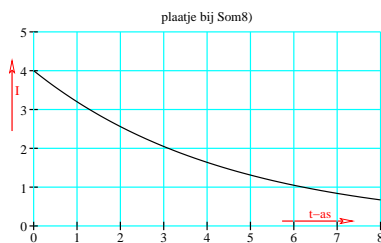
9a) Oud 100% $\xrightarrow{\text{maal } 0,8}$ Nieuw 100% - 20% = 80% $\rightarrow g = 0,8$

9b)

t	0	1	2	3	4	5	6	7	8
I	4	3,2	2,56	2,05	1,64	1,31	1,05	0,84	0,67

9c) $I = 4 \cdot 0,8^t$

9d) $\xrightarrow{\text{Zie tabel } t=4 \Rightarrow I=1,64}$ Na 4 weken



10a)

t	0	$\xrightarrow{+1}$	1
B	1	$\xrightarrow{+0,3}$	1,3

$$\Rightarrow B = 0,3 \cdot t + 1$$

$$\xrightarrow{*1,3} \Rightarrow B = 1 \cdot (1,3)^t \Rightarrow B = (1,3)^t$$

KERN 2 NIEUWE EXPONENTEN

11) $O = 81 \cdot 3^t$ ($t = 0 \rightarrow 1 \text{ jan } 1990$)

11a) $t = 0 \Rightarrow O = 81 \cdot 3^0 \Rightarrow O = 81m^2$

$1/1/1989 \rightarrow t = -1 \Rightarrow O = 81 \cdot 3^{-1} \Rightarrow O = 27m^2$

11b)

t	-3	-2	-1	0	1	2	3
O	3	9	27	81	243	729	2187

$t = -1 \Rightarrow O = 81 \cdot 3^{-1} = 27$

$t = -2 \Rightarrow O = 81 \cdot 3^{-2} = 9$

$t = -3 \Rightarrow O = 81 \cdot 3^{-3} = 3$

11c)

12)

t	4	3	2	1	0	-1	-2	-3	-4
2^t	16	8	4	2	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$

13a) $2^{-5} = \frac{1}{2^5} = \frac{1}{32} = 0.03125$

13b) $5^{-2} = \frac{1}{5^2} = \frac{1}{25}$

13c) $4^{-3} = \frac{1}{4^3} = \frac{1}{64}$

13d) $(0,1)^{-4} = \left(\frac{1}{10}\right)^{-4} = \frac{1}{\left(\frac{1}{10}\right)^4} = \frac{1}{\frac{1}{10^4}} = \frac{10^4}{1} = 10^4 = 10.000$

13e) $3 \cdot 2^{-3} = 3 \cdot \frac{1}{2^3} = 3 \cdot \frac{1}{8} = \frac{3}{8}$

13f) $2 \cdot 3^{-3} = 2 \cdot \frac{1}{3^3} = 2 \cdot \frac{1}{27} = \frac{2}{27}$

13g) $\frac{1}{4} \cdot \left(\frac{1}{2}\right)^{-1} = \frac{1}{4} \cdot \frac{1}{\frac{1}{2}} = \frac{1}{4} \cdot 2 = \frac{1}{2}$

13h) $4 \cdot \left(\frac{1}{4}\right)^{-2} = 4 \cdot \frac{1}{\left(\frac{1}{4}\right)^2} = 4 \cdot \frac{1}{\frac{1}{16}} = 4 \cdot 16 = 64$

14) $H = 4,2 \cdot (0,98)^t$

14a)

$t = 0 \Rightarrow H = 4,2 \cdot 1 = 4,1$

$t = 3 \Rightarrow H = 4,2 \cdot (0,98)^3 \simeq 3,95$

$t = 5 \Rightarrow H = 4,2 \cdot (0,98)^5 \simeq 3,80$

14b)

$t = -1 \Rightarrow H = 4,2 \cdot (0,98)^{-1} \simeq 4,29$

$t = -3 \Rightarrow H = 4,2 \cdot (0,98)^{-3} \simeq 4,46$

$t = -10 \Rightarrow H = 4,2 \cdot (0,98)^{-10} \simeq 5,14$

15) $\rightarrow \text{Oud } 100\% \xrightarrow{\text{maal } 1,0067} \text{Nieuw } 100\% + 0,67\% = 100,67\%$

15a) Groefactor per jaar is 1,0067

15b) $B = 13 \cdot (1,0067)^t$

15c)

1970 $\rightarrow t = 0$

1900 $\rightarrow t = -70 \Rightarrow B = 13 \cdot (1,0067)^{-70} \Rightarrow B \simeq 8,1459 \text{ miljoen}$

Komt niet overeen

Of er is geen exponentiele groei

Of de groefactor is groter

Of er is bijv. een exponentiele met groefactor 1,0067 vanaf 1950

16a) $t = 0 \Rightarrow H = 1$ $t = 1 \Rightarrow H = 2$

$t = \frac{3}{7} \Rightarrow H \simeq 1,35$

16b) Na 1 dag: $\rightarrow t = \frac{1}{7}$

$t = \frac{4}{7} \Rightarrow H \simeq 1,5$

$t = \frac{1}{7} \Rightarrow H \simeq 1,1$

$t = \frac{5}{7} \Rightarrow H \simeq 1,64$

$t = \frac{2}{7} \Rightarrow H \simeq 1,22$

$t = \frac{6}{7} \Rightarrow H \simeq 1,8$

17a) $5^{1,2} \simeq 6,90$

17b) $(1,5)^{-2,5} \simeq 0,36$

17c) $0,8^{-4,5} \simeq 2,73$

17d) $25^{\frac{1}{3}} \simeq 2,92$

17e) $5^{3\frac{1}{4}} \simeq 186,92$

17f) $16^{-\frac{1}{6}} \simeq 0,66$

18) $g = 7$ (per uur)

18a) $B = 100.000 \cdot 7^t$ (t in uren)

18b) $B\left(\frac{1}{4}\right) = 100.000 \cdot 7^{\frac{1}{4}} \simeq 162658$

18c) $B\left(3\frac{1}{2}\right) = 100.000 \cdot 7^{3\frac{1}{2}} \simeq 90749270$

18b) $B\left(8\frac{35}{60}\right) = 100.000 \cdot 7^{8\frac{35}{60}} \simeq 1793734564080 \simeq 1,79 \times 10^{12}$

19) $H = 1000 \cdot (0,9)^t$ (H in cm^3 ; t in dagen $t = 0 \rightarrow$ maandag 12.00 uur)

19a)

Dinsdag 12.00 uur $t = 1$

Dinsdag 20.00 uur $t = 1\frac{8}{24} = 1\frac{1}{3}$

$H\left(1\frac{1}{3}\right) = 1000 \cdot (0,9)^{1\frac{1}{3}} \simeq 868,94m^3$

$H\left(5\frac{1}{2}\right) = 1000 \cdot (0,9)^{5\frac{1}{2}} \simeq 560,19m^3 \xrightarrow{\text{Er is Weggelekt}} 1000 - 560,19 \simeq 439,81m^3$

20a) Na 20 jaar ongeveer $Hfl 9600$, –

$K = 1000 \cdot (1,12)^{20} \simeq 9646,30$

20b) $1000 \cdot (1,12)^t = 4050 \Rightarrow$

$\Rightarrow (1,12)^t = \frac{4050}{1000} = 4,05$

$(1,12)^{12} \simeq 3,90$

$(1,12)^{13} \simeq 4,36$

$(1,12)^{12,5} \simeq 4,12$

Halverwege 2010 kan Zij haar geld opnemen

21) $g = 3$ (per 4 dagen)

21a) $O = 2 \cdot 3^t$

21b) $O = 2 \cdot 3^{2\frac{1}{2}} = 2 \cdot 3^2 \cdot 3^{\frac{1}{2}} = 18\sqrt{3} \simeq 31,18$

21c) $O = 2 \cdot 3^{-\frac{1}{2}} = 2 \cdot \frac{1}{3^{\frac{1}{2}}} = 2 \cdot \frac{1}{\sqrt{3}} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{\sqrt{3}\cdot\sqrt{3}} = \frac{2\sqrt{3}}{3} = \frac{2}{3}\sqrt{3} \simeq 1,15$

$O = 2 \cdot 3^{-\frac{1}{4}} = 2 \cdot \frac{1}{3^{\frac{1}{4}}} = 1,52$

21d) $3^{\frac{1}{4}}$

KERN 3 GRAFIEKEN

22) Oud 100% $\xrightarrow{\text{maal } 0,95}$ Nieuw $100\% - 5\% = 95\%$

22a) $H = 1 \cdot (0,95)^t = (0,95)^t$

22b) $\xrightarrow{\text{Een week heeft}} 7 \cdot 24 = 168 \text{ uren}$

$H = 0,95^{168} \simeq 1,8 \times 10^{-4} \Rightarrow H \simeq 0,00018 \text{mg}$

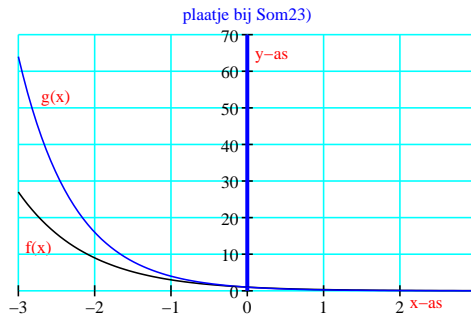
22c) Volgens deze formule niet

23) $f(x) = \left(\frac{1}{3}\right)^x \quad g(x) = \left(\frac{1}{4}\right)^x$

23a)

t		-3	-2	-1	0	1	2	3
$f(x)$		27	9	3	1	$\frac{1}{3}$	$\frac{1}{9}$	$\frac{1}{27}$
$g(x)$		64	16	4	1	$\frac{1}{4}$	$\frac{1}{16}$	$\frac{1}{64}$

23c) Neen



24a) $f(x) = (0,5)^x$

x		-3	-2	-1	0	1	2	3	4
$f(x)$		8	4	2	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$

De vergelijking van de Asymptot $\rightarrow y = 0$

24b) $g(x) = 3 \cdot (0,5)^x$

x		-3	-2	-1	0	1	2	3	4
$g(x)$		24	12	6	3	$1\frac{1}{2}$	$\frac{3}{4}$	$\frac{3}{8}$	$\frac{3}{16}$

De vergelijking van de Asymptot $\rightarrow y = 0$

24c) $h(x) = (0,5)^x + 3$

x		-3	-2	-1	0	1	2	3	4
$h(x)$		11	7	5	4	$3\frac{1}{2}$	$3\frac{1}{4}$	$3\frac{1}{8}$	$3\frac{1}{16}$

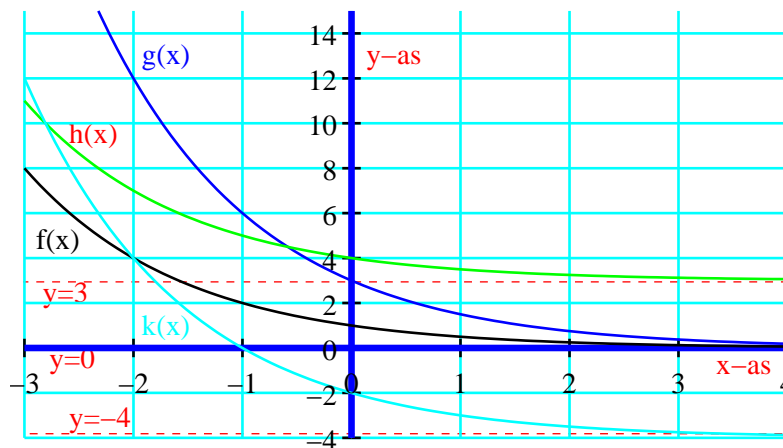
De vergelijking van de Asymptot $\rightarrow y = 3$

24d) $k(x) = 2 \cdot (0,5)^x - 4$

x		-3	-2	-1	0	1	2	3
$k(x)$		13	4	0	-2	-3	$-3\frac{1}{2}$	$-3\frac{3}{4}$

De vergelijking van de Asymptot $\rightarrow y = -4$

plaatje bij Som 24)



25) 20% lost op \rightarrow 80% lost niet op

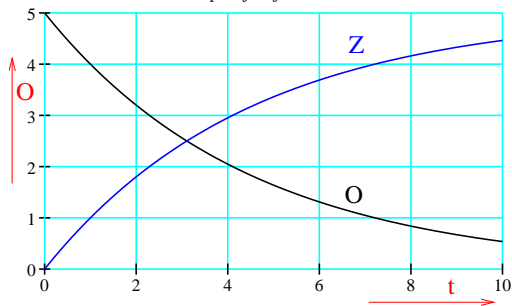
25a) $Z = 5 \cdot 0,8^t$

25b&c)

t	0	1	2	3	4	5	6	7	8	9	10
Z	5	4	3,2	2,56	2,05	1,64	1,31	1,05	0,84	0,67	0,54
O	0	1	1,8	2,44	2,95	3,36	3,69	3,95	4,16	4,33	4,46

O is de opgeloste hoeveelheid zout $H = 5 - 5 \cdot 0,8^t$

25d) $\xrightarrow{\text{Vergelijking van Asymptoot}} y = 5$
 plaatje bij Som 25



26a) t in maanden

Heliumverlies 5% \Rightarrow blijft over 95% \Rightarrow

$\Rightarrow g = 0,95$

$H = 7000 \cdot (0,95)^t$

26b)

t	0	6	12	18	24	36
H	7000	5145,6	3782,5	2780,5	2043,9	1104,5

26c) $H(13,5) = 7000 \cdot (0,95)^{13} \approx 3594 \approx 3500$

26d)

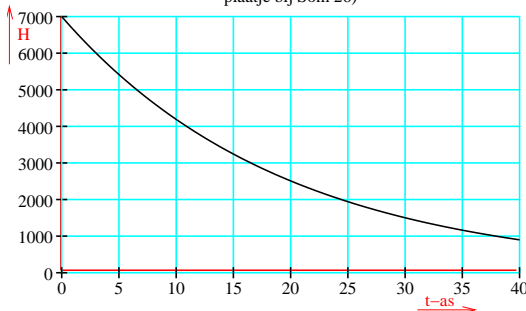
$$\left. \begin{aligned} H(30) &= 7000 \cdot (0,95)^{30} \approx 1502 \\ H(26) &= 7000 \cdot (0,95)^{26} \approx 1845 \\ H(27) &= 7000 \cdot (0,95)^{27} \approx 1752 \\ H(28) &= 7000 \cdot (0,95)^{28} \approx 1665 \end{aligned} \right\} \Rightarrow$$

\Rightarrow Na 28 Maanden

$$\left. \begin{aligned} H(40) &= 7000 \cdot (0,95)^{40} \approx 899,6 \\ H(41) &= 7000 \cdot (0,95)^{41} \approx 854,6 \end{aligned} \right\} \Rightarrow$$

\Rightarrow Na 41 Maanden

plaatje bij Som 26)



27a) Konijn heeft 0,20 meter voorsprong $\left(100 \cdot \left(\frac{1}{2}\right)^9\right)$

a	0	1	2	3	4	5	6	7	8	9	
Hans	0	150	275	387,5	493,75	596,875	698,44	799,22	899,61	899,61	
Konijn	0	100	200	300	400	500	600	700	800	900	1000
Voorsprong	100	50	25	12,5	6,25	3,125	1,56	0,78	0,39	0,20	

$\searrow \times \frac{1}{2} \nearrow$

27b) $V = 100 \cdot \left(\frac{1}{2}\right)^a$ waarbij a is het aantal keren van 100meter verder

$10\text{km} = 10.000\text{meter} = 100 * 100\text{meter verder}$

$V = 100 \cdot \left(\frac{1}{2}\right)^{99} \approx 1,6 \times 10^{-28}$

27c) Neen

28) $f(x) = 2^x$

28a)

$f(0) = 2^0 = 1$

$f(10) = 2^{10} = 1024$

$f(100) = 2^{100} \approx 1,27 \times 10^{30}$

$f(-1) = 2^{-1} = \frac{1}{2}$

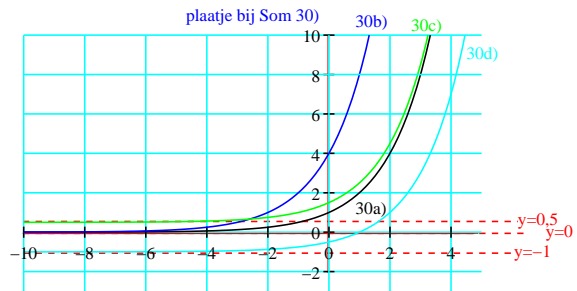
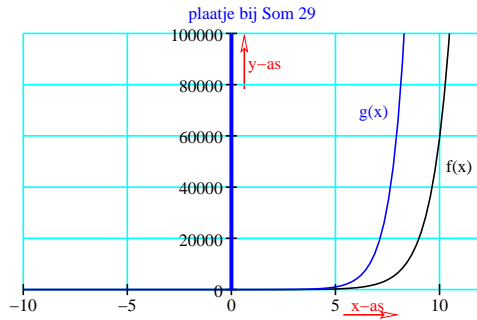
$f(-10) = 2^{-10} = \frac{1}{2^{10}} = \frac{1}{1024}$

$f(-100) = 2^{-100} = \frac{1}{2^{100}} \approx 7,9 \times 10^{-31}$

29a) $f(x) = 3^x$ $g(x) = 4^x$

x	-10	-8	-6	-4	-2	0	2	4	6	8	10
f(x)	0,000017	0,00015	0,0013	0,012	0,111	1	9	81	729	6561	59049
g(x)	0,00000095	0,000015	0,00024	0,004	0,0625	1	16	256	4096	65536	1008576

29c) Neen



30a) $f(x) = 2^x$

x	-3	-2	-1	0	1	2	3	De vergelijking van de Asymptot
f(x)	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8	$y = 0$

30b) $f(x) = 4 \cdot 2^x$

x	-3	-2	-1	0	1	2	3	De vergelijking van de Asymptot
f(x)	$\frac{1}{2}$	1	2	4	8	16	32	$y = 0$

30c) $f(x) = 2^x + \frac{1}{2}$

x	-3	-2	-1	0	1	2	3	De vergelijking van de Asymptot
f(x)	$\frac{5}{8}$	$\frac{3}{4}$	1	$1\frac{1}{2}$	$2\frac{1}{2}$	$4\frac{1}{2}$	$8\frac{1}{2}$	$y = \frac{1}{2}$

30d) $f(x) = \frac{1}{2} \cdot 2^x - 1$

x	-3	-2	-1	0	1	2	3	De vergelijking van de Asymptot
f(x)	$-\frac{15}{16}$	$-\frac{7}{8}$	$-\frac{3}{5}$	$-\frac{1}{2}$	0	1	3	$y = -1$

31) Groei: $\xrightarrow{10\% \text{ per week bij } 20^\circ C} g = 1,1$
(t in weken)

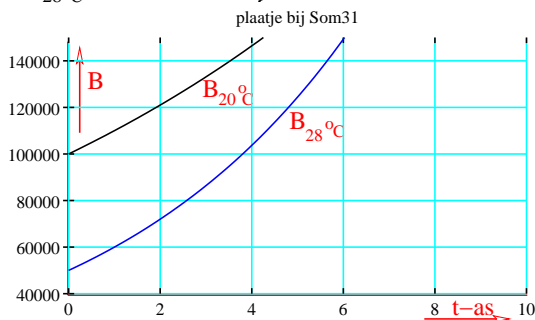
31b) $B_{20^\circ C} = 100.000 \cdot (1,1)^t$

x	0	1	2	3
$B_{20^\circ C}$	100.000	110.000	121.000	133.100

31c) $B_{28^\circ C} = 50.000 \cdot (1,2)^t$

x	0	1	2	3
$B_{28^\circ C}$	50.000	60.000	72.000	86.400

$B_{20^\circ C}(8) \simeq 214.359$
 $B_{28^\circ C}(8) \simeq 214.991$ } $\Rightarrow Na \pm 8 \text{ weken}$



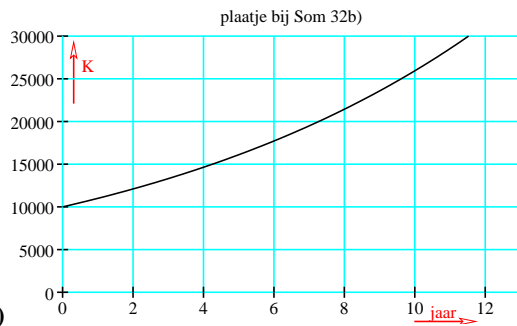
32a) Iets meer dan 7 jaar

$$\left. \begin{array}{l} 10^4 \cdot (1,1)^7 \simeq 19.478 \\ 10^4 \cdot (1,1)^{7\frac{1}{12}} \simeq 19.641,6 \\ 10^4 \cdot (1,1)^{7\frac{2}{12}} \simeq 19.799,2 \\ 10^4 \cdot (1,1)^{7\frac{3}{12}} \simeq 19.957,1 \\ 10^4 \cdot (1,1)^{7\frac{4}{12}} \simeq 20.116 \\ 20.116 \xrightarrow{\text{dus na}} 7\frac{1}{3} \text{ jaar} \end{array} \right\} \Rightarrow \text{Klopt}$$

32d) Geen van beiden is voordeliger

$$10.000 \cdot 1,08^9 \cdot 1,12^4 \simeq 31454,7$$

$$10.000 \cdot 1,12^4 \cdot 1,08^9 \simeq 31454,7$$



32b)

KERN 4 VERGELIJKINGEN & ONGELIJKHEDEN

33a) $50 \cdot (1,065)^{11} \simeq 100$

33b) $12.500 \cdot (1,065)^{11} \simeq 24.98 \rightarrow$ niet helemaal juist

33c) $B \cdot 1,08^t = 2B \Rightarrow 1,08^t = \frac{2B}{B} \Rightarrow 1,08^t = 2$

$$\left. \begin{array}{l} 1,08^8 \simeq 1,85 \\ 1,08^9 \simeq 2,2 \end{array} \right\} \rightarrow \text{Dus 9 jaar}$$

34a)

$$\left. \begin{array}{l} t=0 \Rightarrow y=1 \quad \downarrow \times 2 \\ t=1 \Rightarrow y=2 \quad \downarrow \times 2 \\ t=2 \Rightarrow y=4 \end{array} \right\} \Rightarrow f(t) = 2^t$$

34b) $2^t = 7$

t	y ₁	y ₂	y ₁ - y ₂
2,7	6,498	7	-0,502
2,8	6,96	7	-0,036
2,9	7,46	7	+0,46

$\xrightarrow{\text{Dus}} t \simeq 2,8$

35a) Neem 1960 als $t = 0$

$W = 3 \cdot (1,019)^t$ W in miljarden

1990 $\Rightarrow t = 30 \rightarrow W(30) = 3 \cdot (1,019)^{30} \simeq 5,28 \text{ mil jard}$

2000 $\Rightarrow 5,28 \cdot (1,015)^{10} \simeq 6,12 \text{ mil jard}$

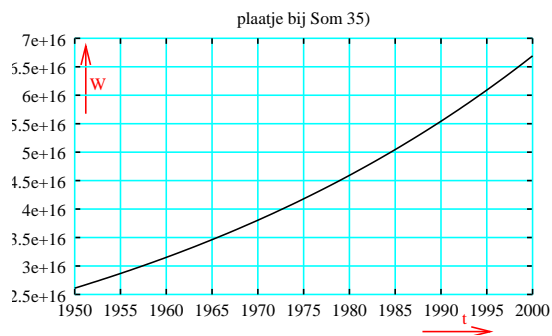
35c)

$$\left. \begin{array}{l} W(15) = 3 \cdot (1,019)^{15} \simeq 3,98 \text{ mil jard} \\ W(16) = 3 \cdot (1,019)^{16} \simeq 4,05 \text{ mil jard} \end{array} \right\} \Rightarrow$$

$\xrightarrow{\text{Dus in}} 1976$

$$\left. \begin{array}{l} W(30) = 3 \cdot (1,019)^{30} \simeq 5,28 \text{ mil jard} \\ 5,28 \cdot 1015^7 \simeq 5,86 \text{ mil jard} \\ 5,28 \cdot 1015^8 \simeq 5,94 \text{ mil jard} \\ 5,28 \cdot 1015^9 \simeq 6,03 \text{ mil jard} \end{array} \right\} \Rightarrow$$

$\xrightarrow{\text{Dus in}} 1999$



36a)

t	0	1	2	3	4	6	8	10
T	75	66,75	59,74	53,78	48,71	40,74	34,99	30,83

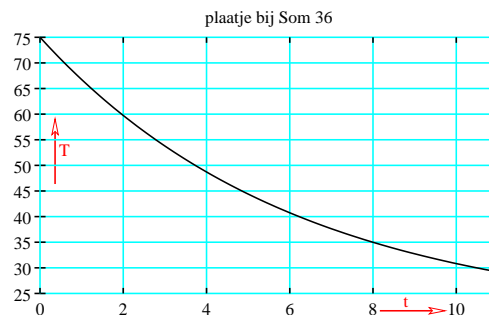
$T = 20 + 55 \cdot (0,85)^t$

T in $^{\circ}C$ en t tijd in minuten

36b) $75^{\circ}C \quad 20^{\circ}C$

$20 + 55 \cdot (0,85)^{100} \simeq 20^{\circ}C$

36c) ± 2 minuten



37)

x	y ₁ = 2 · 5 ^x	y ₂ = 3	y ₁ - y ₂
0,2	2,76	3	-0,24
0,3	3,24	3	-0,24

$\xrightarrow{\text{Dus}} 2 \cdot 5^x < 3 \Rightarrow x < 0,2$

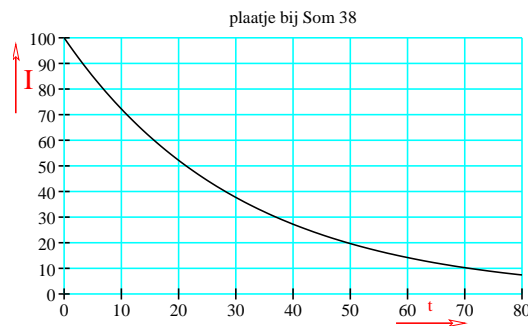
$\xrightarrow{\text{wel iets kleiner dan } y_1}$

38) $I = 100 \cdot (0,968)^t$

t	0	1	2	3	4	5	6	8	10	14	18	20	25	30
I	100	96,8	93,7	90,7	87,8	84,99	82,3	77,1	72,2	63,4	55,69	52,2	44,35	37,69

$100 \cdot (0,968)^{12} \simeq 67,7$

$100 \cdot (0,968)^{11} \simeq 69,9 \xrightarrow{\text{Dusna}} 11 \text{ jaar}$



39a) $x = 1 \Rightarrow 3^1 + 1 = 4 \rightarrow 3^x + 1 > 4 \Rightarrow x > 1$

39b) $5^{-2} = 0,04 \xrightarrow{\text{uitleg}}$

$0,04 = \frac{4}{100} = \frac{1}{25} = \frac{1}{5^2} = 5^{-2}$

$5^x < 0,04 \Rightarrow x < -2$

39c)

$$\left. \begin{array}{l} 3^{0,6} + 1 \simeq 2,93 \\ 3^{0,7} + 1 \simeq 3,16 \\ 3^{0,63} + 1 \simeq 2,998 \end{array} \right\} \Rightarrow 3^x + 1 \leq 3 \Rightarrow x \leq 0,63$$

x	$y_1 = 3^x + 1$	$y_2 = 5^x$	$y_1 - y_2$	$\Delta(y_1 - y_2)$
0,5	2,73	2,24	+0,49	
0,6	2,93	2,63	+0,3	+0,19
0,7	3,16	3,09	+0,07	+0,23
0,71	3,18	3,14	+0,04	+0,03
0,72	3,31	3,19	+0,02	+0,02
0,8	3,41	3,62	-0,21	-0,23

39d)

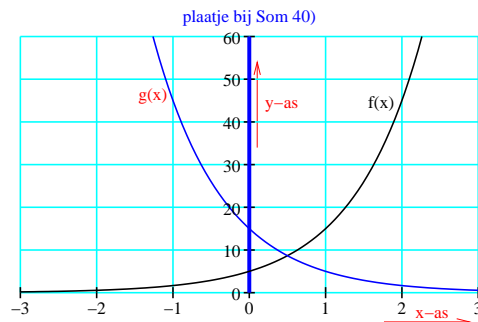
$3^x + 1 \geq 5^x$
 $y_1 \geq y_2 \Rightarrow x \leq 0,72$

40) $f(x) = 5 \cdot 3^x$ $g(x) = 15 \cdot (\frac{1}{3})^x$

x	-2	-1	0	0,4	0,5	1	2
$f(x)$	$\frac{5}{9}$	$\frac{5}{3}$	5	7,76	8,66	15	45
$g(x)$	135	45	15	9,67	8,66	5	$\frac{15}{9}$

40b) $g(x) \geq 5 \Rightarrow x \leq 1$

40c) $f(x) < g(x) \xrightarrow{\text{zietabel}} x < 0,5$

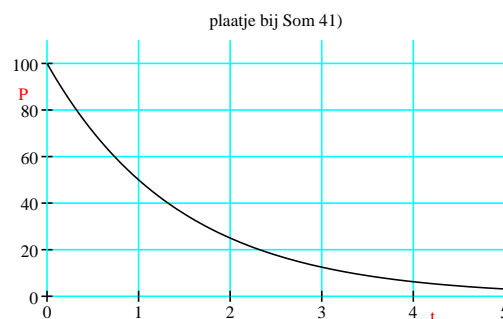


41a) $\frac{1}{2}$

41b) $P = 100 \cdot (\frac{1}{2})^t$ in 5730 jaren

41c) $t = 5 \Rightarrow P = 100 \cdot (\frac{1}{2})^5 \Rightarrow P = 3,125$

Dus: $5 * 5730 = 28650$ jaar oud



42a) Afname per uur 29% $\xrightarrow{\text{blijft over: } 100\% - 29\%}$ = 71% $\Rightarrow g = 0,71$

42b) $H = 800 \cdot (0,71)^t$ H in milligram ; t in uren

42d)

$$\left. \begin{array}{l} H(3) = 800 \cdot (0,71)^3 \simeq 286,3 \\ H(2,8) = 800 \cdot (0,71)^{2,8} \simeq 306,6 \\ H(2,9) = 800 \cdot (0,71)^{2,9} \simeq 296,31 \end{array} \right\} \xrightarrow{\text{Dusna}} \pm 2 \text{ uur en } 50 \text{ minuten}$$

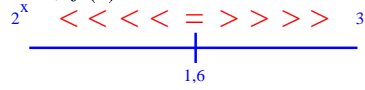
$$\left. \begin{array}{l} H(10) = 800 \cdot (0,71)^{10} \simeq 26,04 \\ H(15) = 800 \cdot (0,71)^{15} \simeq 4,70 \\ H(16) = 800 \cdot (0,71)^{16} \simeq 3,3 \\ H(17) = 800 \cdot (0,71)^{17} \simeq 2,4 \\ H(18) = 800 \cdot (0,71)^{18} \simeq 1,68 \end{array} \right\} \xrightarrow{\text{Dusna}} \pm 17 \text{ uur}$$

GRAFISCHE REKENMACHINE

G1a) $f(x) = 2^x$ $D_f = [-3; 3]$

G1b) $f(x) = 3 \Rightarrow x \approx 1,6$

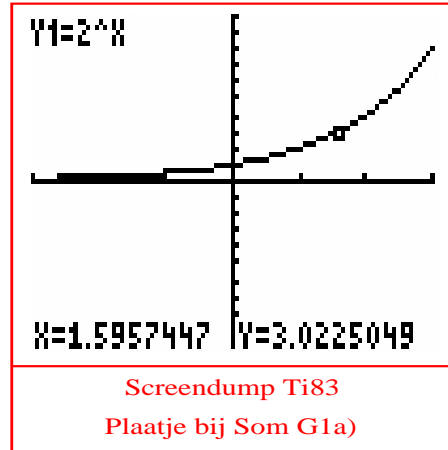
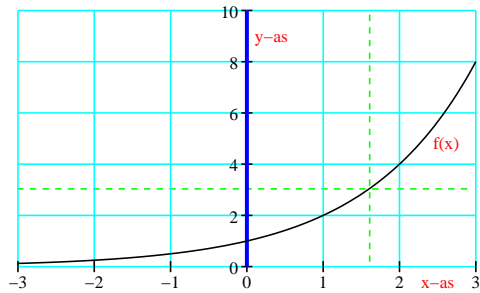
G1c) $f(x) < 3$



$x = 0 \Rightarrow 2^0 = 1 < 3$

$f(x) < 3 \Rightarrow x < 1,6$

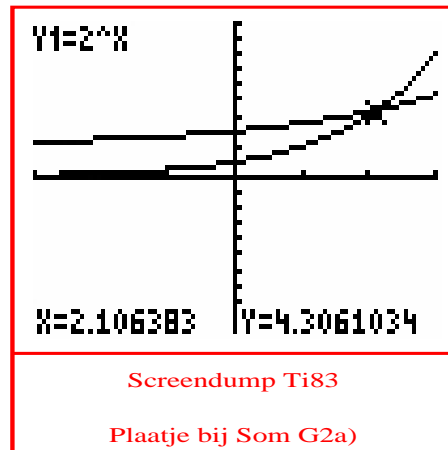
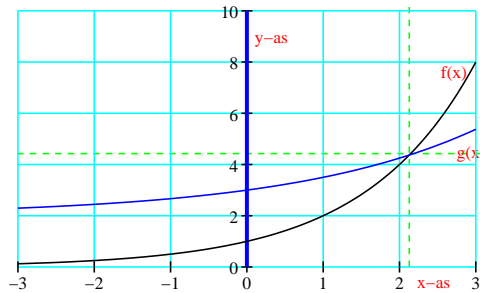
plaatje bij Som G1a)



G2a) $f(x) = 2^x$ $g(x) = 1,5^x + 2$

$x \approx 2,13; y \approx 4,37$

plaatje bij Som G2a)



G3a) $y_1 = 2^x$ $y_2 = 5 \cdot 0,5^x + 4$

t	0	1	2	3	4	5
$y_1 = 2^x$	1	2	4	8	16	32
$y_2 = 5 \cdot 0,5^x + 5$	9	6,5	5,25	4,625	4,3125	4,16

$y_1 = y_2$ *De x-waarde ligt tussen* \rightarrow 2 en 3

G3b)

t	2	2,1	2,2	2,3	2,4
$y_1 = 2^x$	4	4,287	4,595	4,925	5,278
$y_2 = 5 \cdot 0,5^x + 5$	5,25	5,166	5,088	5,015	4,947
$y_1 - y_2$			-0,493	-0,09	+0,331

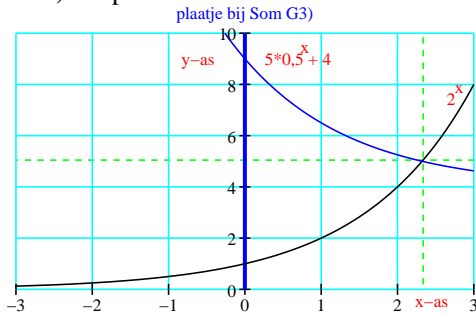
Dus $\rightarrow x \approx 2,3$

G3c)

t	2,4	2,31	2,32	2,33
$y_1 = 2^x$	4,925	4,959	4,993	5,028
$y_2 = 5 \cdot 0,5^x + 5$	5,015	5,008	5,001	4,994
$y_1 - y_2$			-0,008	+0,034

Dus $\rightarrow x \approx 2,32$

G3d) Klopt



G4a)

$$\left. \begin{array}{l} (0; 5) \quad (2; 8) \quad f(x) = a^x + b \\ 5 = a^0 + b \Rightarrow 5 = 1 + b \quad b = 4 \\ 8 = a^2 + b \end{array} \right\} \Rightarrow 8 = a^2 + 4 \Rightarrow a^2 = 4 \Rightarrow a = 2 \xrightarrow{a=2 \& b=4} f(x) = 2^x + 4$$

$$\left. \begin{array}{l} (0; 1) \quad (1; 3) \quad f(x) = a^x + b \\ 1 = a^0 + b \Rightarrow 1 = 1 + b \quad b = 0 \\ 3 = a^1 + b \end{array} \right\} \Rightarrow a = 3 \xrightarrow{a=3 \& b=0} f(x) = 3^x$$

$$\left. \begin{array}{l} (0; 0) \quad (2; 3) \quad f(x) = a^x + b \\ 0 = a^0 + b \Rightarrow 0 = 1 + b \quad b = -1 \\ 3 = a^2 + b \end{array} \right\} \Rightarrow 3 = a^2 - 1 \Rightarrow a = 2 \xrightarrow{a=2 \& b=-1} f(x) = 2^x - 1$$

$$\left. \begin{array}{l} (0; 2) \quad (-2; 5) \quad f(x) = a^x + b \\ 2 = a^0 + b \Rightarrow 2 = 1 + b \quad b = 1 \\ 5 = a^{-2} + b \end{array} \right\} \Rightarrow 5 = a^{-2} + 1 \Rightarrow a^2 = \frac{1}{4} \Rightarrow a = \frac{1}{2}$$

$$\xrightarrow{a=\frac{1}{2} \& b=1} f(x) = \left(\frac{1}{2}\right)^x + 1 \Rightarrow f(x) = 2^{-x} + 1$$

$$\left. \begin{array}{l} (-1; 2) \quad (0; -1) \quad f(x) = a^x + b \\ -1 = a^0 + b \Rightarrow -1 = 1 + b \quad b = -2 \\ 2 = a^{-1} + b \end{array} \right\} \Rightarrow 2 = a^{-1} - 2 \Rightarrow a^{-1} = 4 \Rightarrow a = \frac{1}{4}$$

$$\xrightarrow{a=\frac{1}{4} \& b=-2} f(x) = \left(\frac{1}{4}\right)^x - 2 \Rightarrow f(x) = 4^{-x} - 2$$

$$\left. \begin{array}{l} (0; 5) \quad (-1; 6) \quad f(x) = a^x + b \\ 5 = a^0 + b \Rightarrow 5 = 1 + b \quad b = 4 \\ 6 = a^{-1} + b \end{array} \right\} \Rightarrow 6 = a^{-1} + 4 \Rightarrow a^{-1} = 2 \Rightarrow \frac{1}{a} = 2 \Rightarrow a = \frac{1}{2}$$

$$\xrightarrow{a=\frac{1}{2} \& b=4} f(x) = \left(\frac{1}{2}\right)^x + 4 \Rightarrow f(x) = 2^{-x} + 4$$

1^e Snijpunt (1,84; 7,59)

G4c) 2^e Snijpunt (1,27; 1,42)

3^e Snijpunt (-1,38; 7,0)

G5) $M = b \cdot 2^{-ct}$ M in mg ; t in dagen; b = 24mg

G5a) $c = 1 \Rightarrow M = 24 \cdot 2^{-t}$

t	0	1	2	3	4	5	6	7
M=24 · 2 ^{-t}	24	12	6	4	1,5	0,75	0,375	0,1875

$$t = \frac{100}{24} \Rightarrow M = 24 \cdot 2^{-\frac{100}{24}} \simeq 1,34 \text{mg}$$

G5b)

$$\left. \begin{array}{l} (0; 24) \quad \frac{100}{24} = 4\frac{4}{6} = 4\frac{1}{6} \\ (4\frac{1}{6}; 18) \\ M = 24 \cdot 2^{-ct} \end{array} \right\} \Rightarrow 18 = 24 \cdot 2^{-c \cdot 4\frac{1}{6}} \Rightarrow 2^{-4\frac{1}{6}c} = 0,75 \Rightarrow c \simeq +0,0996$$

G5c) $24 \cdot 2^{-0,0996t} = 12 \Rightarrow 2^{-0,0996t} = \frac{1}{2} \Rightarrow 2^{-0,0996t} = 2^{-1} \Rightarrow 0,0996t = 1 \Rightarrow t \simeq 10 \leftarrow \text{dus ongeveer 10 dagen}$

G6) $1650 \rightarrow 0,9 \times 10^9$ mensen Groei 0,3% per jaar

G6a) $1650 \rightarrow t = 0 \rightarrow g = 0,003$

$A = 0,9 \cdot (1,003)^t$ A in miljarden en t in jaren

t	1650	1675	1700	1725	1750	1775	1800	1825	1850	1875	1900	1925	1950	1975	2000
A	0,9	0,97	1,05	1,13	1,21	1,31	1,41	1,52	1,64	1,77	1,90	2,05	2,21	2,38	2,57

G6c) $t = 325 \Rightarrow A = 0,9 \cdot (1,003)^{325} \simeq 2,38$ mil jard

G6d)

$$\left. \begin{array}{l} (0; 0,9) \\ (325; 3,7) \end{array} \right\} \Rightarrow A = 0,9 \cdot g^t$$

$$3,7 = 0,9 \cdot g^{325} \Rightarrow \frac{3,7}{0,9} = g^{325} \Rightarrow g^{325} \simeq 4,11 \Rightarrow \sqrt[325]{4,11} \simeq g \Rightarrow g \simeq 1,0044$$

jaar 2000 $\rightarrow t = 350 \Rightarrow A = 0,9 \cdot 1,0044^{350} \simeq 4,18$ mil jard

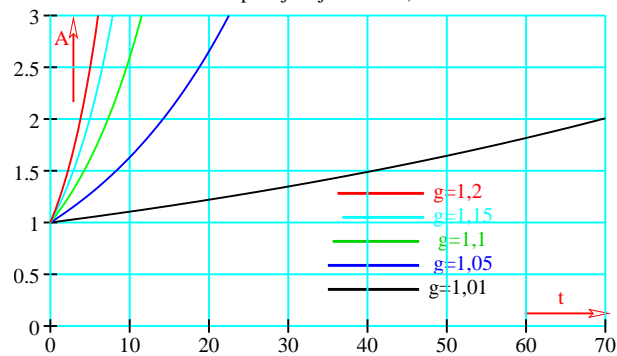
G6e) $A = (1,02)^t$

$t \simeq 35 \rightarrow 1,02^{35} \simeq 2$

G6f)

$$\begin{array}{l} 1\% \Rightarrow \left. \begin{array}{l} g = 1,01 \\ A = 1,01^t \end{array} \right\} \Rightarrow \text{verdubbelingstijd} \simeq 69,8 \\ 5\% \Rightarrow \left. \begin{array}{l} g = 1,05 \\ A = 1,05^t \end{array} \right\} \Rightarrow \text{verdubbelingstijd} \simeq 14,3 \\ 10\% \Rightarrow \left. \begin{array}{l} g = 1,1 \\ A = 1,1^t \end{array} \right\} \Rightarrow \text{verdubbelingstijd} \simeq 7,3 \\ 15\% \Rightarrow \left. \begin{array}{l} g = 1,15 \\ A = 1,15^t \end{array} \right\} \Rightarrow \text{verdubbelingstijd} \simeq 5 \\ 20\% \Rightarrow \left. \begin{array}{l} g = 1,2 \\ A = 1,2^t \end{array} \right\} \Rightarrow \text{verdubbelingstijd} \simeq 3,8 \end{array}$$

plaatje bij Som G6f)



G6g) Groeifactor $\frac{\text{verdubbelingstijd}}{\rightarrow} = 2$

DOORWERKING

D1a) $0 \leq t \leq 3$

D1b)

$$\left. \begin{array}{l} (0; 6) \\ (3; 20) \end{array} \right\} \Rightarrow l = 6 \cdot g^t \rightarrow 20 = 6 \cdot g^3 \Rightarrow g^3 = 3\frac{1}{3} \Rightarrow g = \sqrt[3]{3\frac{1}{3}} \simeq 1,5$$

$$l = 6 \cdot 1,5^t$$

D1c) $4 \leq t \leq 6$

D1d)

$$\left. \begin{array}{l} (4; 23,9) \\ (6; 28,1) \end{array} \right\} \Rightarrow \frac{\Delta l}{\Delta t} = \frac{28,1-23,9}{6-4} = \frac{4,2}{2} = 2,1 \Rightarrow$$

$$\Rightarrow \left. \begin{array}{l} l = 2,1 \cdot t + b \\ (4; 23,9) \end{array} \right\} \Rightarrow 23,9 = 4 \cdot 2,1 + b \Rightarrow b = 15,5 \rightarrow l = 2,1t + 15,5$$

D1e) in de 3^e week

$$\left. \begin{array}{l} (3; 20) \\ (2; 13,3) \end{array} \right\} \Rightarrow 6,9 \text{ cm/week}$$

D1f) Van 13,3cm naar 20cm dus;

$$\frac{20-13,3}{31} * 100 \simeq 21,6\% \text{ van zijn totale lengte kwam er in de derde week bij.}$$

D2) 1 november 1986 ($t = 0$) 35 ton Bad Honnef

per 10 km \rightarrow afname 2% $\xrightarrow{\text{blijft over}}$ 98%

Bonn: 14,7 $\mu\text{g/l}$

D2a)

$$\frac{a \text{ (in 10 km)}}{G} \parallel \begin{array}{c} 0 \\ 15 \end{array} \xrightarrow{\times 0,98} \begin{array}{c} 1 \\ 14,8 \end{array} \rightarrow \frac{14,7}{0,98} = 15 \mu\text{g/liter}$$

D2b) 0,98 is de groeifactor per 10 jaar

$$G = 15 \cdot 0,98^{0,16} \xrightarrow{\text{b in jaren}} \text{Antwoordenboek}$$

D2c) Rotterdam: 350km $\Rightarrow a = 35 \rightarrow G = 15 \cdot 0,98^{35} \simeq 7,4 \leftarrow \text{winning werd gestopt}$

D2d) $5 = 15 \cdot 0,98^a \Rightarrow \frac{1}{3} = 0,98^a \Rightarrow a \simeq 54,4 \Rightarrow 544 \text{ km} \leftarrow \text{Stroomafwaarts}$

D2e) Er zijn meer factoren (dan alleen de zijrivieren) die het gifgehalte beïnvloeden.

Bijvoorbeeld de hoeveelheid neerslag, de temperatuur, zuiverings installaties.

D3a) $I(x) = I(0) \cdot 2^{-ax}$; a is een materiaalconstante

$$\left. \begin{array}{l} \text{Lood : } x = 10 \rightarrow I(10) = \frac{1}{2} I(0) \\ I(10) = I(0) \cdot 2^{-10a} \end{array} \right\} \Rightarrow \frac{1}{2} \cdot I(0) = I(0) \cdot 2^{-10a} \Rightarrow \frac{1}{2} = 2^{-10a} \Rightarrow 2^{-1} = 2^{-10a} \Rightarrow$$

$$\Rightarrow 10a = 1 \Rightarrow a = 0,1$$

D3b) $2^{-0,02 \cdot 98} \simeq 0,986$

D3c)

$$\left. \begin{array}{l} I(30) = \frac{1}{2} I(0) \\ I(30) = I(0) \cdot 2^{-30a} \end{array} \right\} \Rightarrow \frac{1}{2} \cdot I(0) = I(0) \cdot 2^{-30a} \Rightarrow \frac{1}{2} = 2^{-30a} \Rightarrow 2^{-1} = 2^{-30a} \Rightarrow 30a = 1 \Rightarrow$$

$$\Rightarrow a = \frac{1}{30}$$

$$\left. \begin{array}{l} \text{Stel } I(3) = 2 \\ I(x) = I(0) \cdot 2^{-\frac{1}{30}x} \end{array} \right\} \Rightarrow 2 = I(0) \cdot 2^{-\frac{1}{30} \cdot 30} \Rightarrow 2 = I(0) \cdot 2^{-1} \Rightarrow 2 = I(0) \cdot 2^{-1} \Rightarrow I(0) = 4 \rightarrow$$

$$I(60) = 4 \cdot 2^{-\frac{1}{30} \cdot 60} = 4 \cdot 2^{-2} = \frac{4}{4} = 1$$

$$I(90) = 4 \cdot 2^{-\frac{1}{30} \cdot 90} = 4 \cdot 2^{-3} = \frac{4}{8} = \frac{1}{2}$$

$$I(120) = 4 \cdot 2^{-\frac{1}{30} \cdot 120} = 4 \cdot 2^{-4} = \frac{4}{32} = \frac{1}{8}$$

