hoofdstuk

09 Diagnostische toets

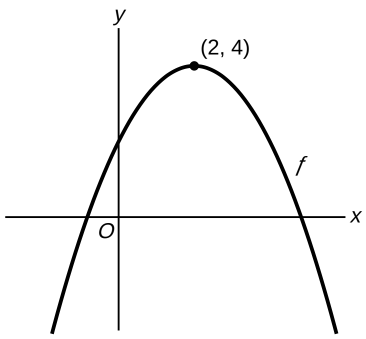
bladzijde 160

1 a $y = 0{,}5x^2$
$\downarrow$ translatie $(3, 6)$
$y = 0{,}5(x - 3)^2 + 6$

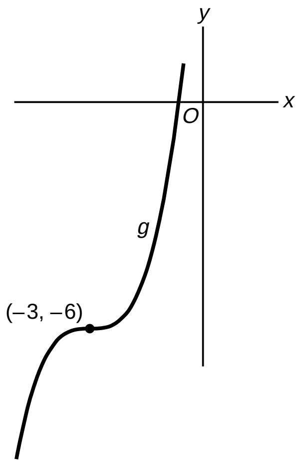
b $y = 0{,}5x^2$
$\downarrow$ translatie $(-2, 5)$
$y = 0{,}5(x + 2)^2 + 5$

c $y = 0{,}5x^2$
$\downarrow$ translatie $(5, 0)$
$y = 0{,}5(x - 5)^2$

2 a



b



3 a $y = 2x^2$
$\downarrow$ verm. x-as met 2,5
$y = 5x^2$
$\downarrow$ translatie $(-2, 3)$
$y = 5(x + 2)^2 + 3$
De top is $(-2, 3)$.

b $y = 2x^2$
$\downarrow$ translatie $(-2, 3)$
$y = 2(x + 2)^2 + 3$
$\downarrow$ verm. x-as met 2,5
$y = 2{,}5\left(2(x + 2)^2 + 3\right)$ ofwel $y = 5(x + 2)^2 + 7{,}5$
De top is $(-2;\ 7{,}5)$.

4 a $y = \sqrt{x}$
$\downarrow$ translatie $(2, 2)$
$y = 2 + \sqrt{x - 2}$

$y = \sqrt{x}$
$\downarrow$ verm. x-as met 3
$y = 3\sqrt{x}$
$\downarrow$ translatie $(1, 2)$
$y = 3\sqrt{x - 1} + 2$

b $D_f = [2, \rightarrow\rangle$, $B_f = [2, \rightarrow\rangle$, $D_g = [1, \rightarrow\rangle$, $B_g = [2, \rightarrow\rangle$

5 a $y = \dfrac{1}{x}$
$\downarrow$ translatie $(-2, 4)$
$y = \dfrac{1}{x + 2} + 4$

b De asymptoten zijn $x = -2$ en $y = 4$.

6 a $g(x) = \dfrac{3x+2}{x+2}$

noemer $= 0$ geeft $x + 2 = 0$

$x = -2$

De verticale asymptoot is de lijn $x = -2$.

$\left.\begin{array}{l} g(100) \approx 2{,}96 \\ g(1000) \approx 2{,}996 \end{array}\right\}$ De horizontale asymptoot is de lijn $y = 3$.

$h(x) = \dfrac{\frac{1}{2}x - 2}{\frac{1}{2}x + 2}$

noemer $= 0$ geeft $\frac{1}{2}x + 2 = 0$

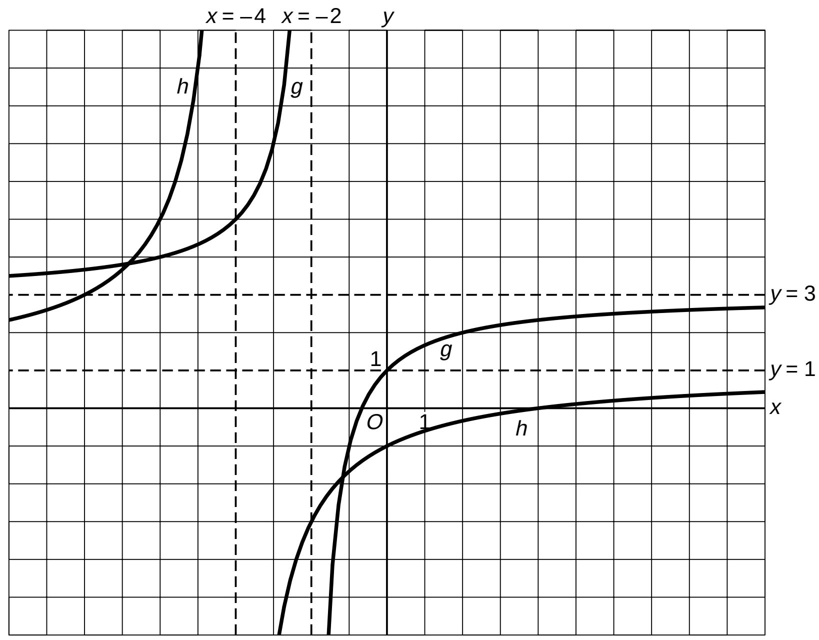
$\frac{1}{2}x = -2$

$x = -4$

De verticale asymptoot is de lijn $x = -4$.

$\left.\begin{array}{l} h(100) \approx 0{,}923 \\ h(1000) \approx 0{,}992 \end{array}\right\}$ De horizontale asymptoot is de lijn $y = 1$.

x	−10	−8	−6	−4	−2	0	2	4	6	8	10
$g(x)$	3,5	3,7	4	5	–	1	2	2,3	2,5	2,6	2,7
$h(x)$	2,3	3	5	–	−3	−1	−0,3	0	0,2	0,3	0,4



b Voer in $y_1 = (3x+2)/(x+2)$ en $y_2 = (\frac{1}{2}x - 2)/(\frac{1}{2}x + 2)$.
De optie intersect geeft $x \approx -6{,}83$ en $x \approx -1{,}17$.
Aflezen $g(x) \leq h(x)$ geeft $-6{,}83 \leq x < -4 \;\vee\; -2 < x \leq -1{,}17$.

7 Voer in $y_1 = (4 - x)/(x + 1)$

$y_2 = \text{nDeriv}(y_1, x, x)$ of $y_2 = \text{d}/\text{d}x(y_1, x)$

$y_3 = -5$

De optie intersect met y_2 en y_3 geeft $x = -2$ en $x = 0$.

$\left.\begin{array}{r} k_1\colon y = -5x + b \\ (-2, -6) \end{array}\right\}$ $\begin{array}{l} -6 = -5 \cdot -2 + b \\ -6 = 10 + b \\ -16 = b \end{array}$

Dus $k_1\colon y = -5x - 16$.

$\left.\begin{array}{r} k_2\colon y = -5x + b \\ (0, 4) \end{array}\right\}$ Dus $k_2\colon y = -5x + 4$.

8 $f(x) = -x^2 + x + 6$

$\downarrow$ verm. y-as met 4

$g(x) = -(\frac{1}{4}x)^2 + \frac{1}{4}x + 6 = -\frac{1}{16}x^2 + \frac{1}{4}x + 6$

9 $f(x) = 4$ geeft $-x^2 + x + 6 = 4$

$-x^2 + x + 2 = 0$

$x^2 - x - 2 = 0$

$(x+1)(x-2) = 0$

$x = -1 \lor x = 2$

De grafiek van f gaat door de punten $(-1, 4)$ en $(2, 4)$.

Dus $p = \frac{6}{2} = 3$.

Dus $g(x) = -(\frac{1}{3}x)^2 + \frac{1}{3}x + 6$ ofwel $g(x) = -\frac{1}{9}x^2 + \frac{1}{3}x + 6$

10 a k is in kwartieren en t in uren, dus $k = 4t$.

Dit geeft $V = 13 \cdot 0{,}78^{4t}$.

$0{,}78^4 \approx 0{,}37$, dus $V = 13 \cdot 0{,}37^t$

b k is in kwartieren en m in minuten, dus $k = \frac{1}{15}m$.

Dit geeft $V = 13 \cdot 0{,}78^{\frac{1}{15}m}$.

$0{,}78^{\frac{1}{15}} \approx 0{,}98$, dus $V = 13 \cdot 0{,}98^m$

11 a $y = 3^{\frac{1}{2}x-1} = 3^{\frac{1}{2}x} \cdot 3^{-1} = \left(10^{\log(3)}\right)^{\frac{1}{2}x} \cdot \frac{1}{3} = \frac{1}{3} \cdot 10^{\frac{1}{2}x\log(3)} \approx \frac{1}{3} \cdot 10^{0{,}239x}$

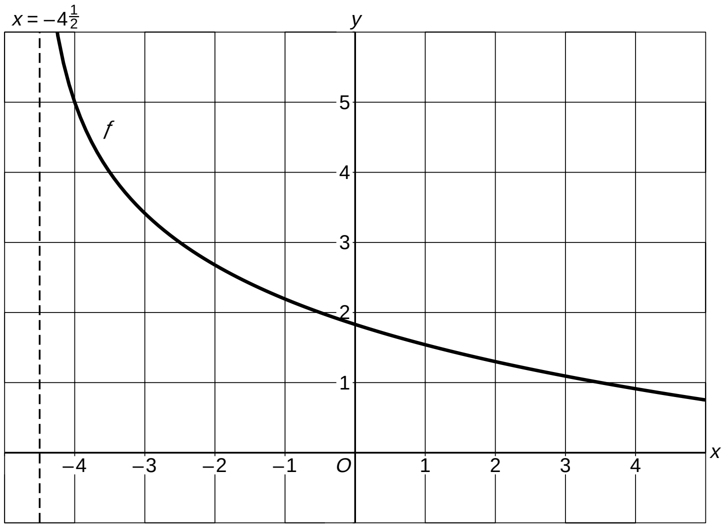
b $y = 52^{0{,}6x+3} = 52^{0{,}6x} \cdot 52^3 = \left(10^{\log(52)}\right)^{0{,}6x} \cdot 140\,608 = 140\,608 \cdot 10^{0{,}6x\log(52)} \approx 140\,608 \cdot 10^{1{,}030x}$

12 a Verticale asymptoot als $2x + 9 = 0$, dus $x = -4\frac{1}{2}$ is verticale asymptoot.

Voer in $y_1 = 5 - \log(2x+9)/\log(2)$.

Tabel

x	-4	-3	-2	-1	0	2	4	6
$f(x)$	5	3,4	2,7	2,2	1,8	1,3	0,9	0,6



b Los op $5 - {}^2\log(2x+9) = 0$

${}^2\log(2x+9) = 5$

$2x + 9 = 2^5$

$2x = 23$

$x = 11\frac{1}{2}$

Aflezen $f(x) \geq 0$ geeft $-4\frac{1}{2} < x \leq 11\frac{1}{2}$.

c $f(0) \approx 1{,}83$

Aflezen $x \leq 0$ geeft $f(x) \geq 1{,}83$.

13 a $AB\colon y = ax + b$ met $a = \dfrac{\Delta y}{\Delta x} = \dfrac{1{,}5 - 4{,}5}{3 - 1} = -1{,}5$

$$\left.\begin{array}{l}\text{Dus } AB\colon y = -1{,}5x + b\\ A\,(1;\ 4{,}5)\end{array}\right\}\begin{array}{l}4{,}5 = -1{,}5 \cdot 1 + b\\ 4{,}5 = -1{,}5 + b\\ b = 6\end{array}$$

Dus $AB\colon y = -1{,}5x + 6$.

De lengte van CD is $L(p) = 0{,}5p(p-4)^2 - (-1{,}5p + 6)$

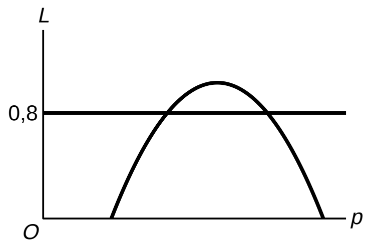
$= 0{,}5p(p-4)^2 + 1{,}5p - 6$.

Voer in $y_1 = 0{,}5x(x-4)^2 + 1{,}5x - 6$.

De optie maximum geeft $x \approx 1{,}78$ en $y \approx 1{,}06$.

Dus de maximale lengte is ongeveer 1,06.

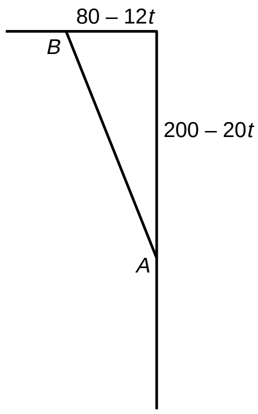
b Voer in $y_2 = 0{,}8$.



De optie intersect geeft $x \approx 1{,}38$ en $x \approx 2{,}27$.

Aflezen $L > 0{,}8$ voor $1{,}38 < p < 2{,}27$.

14 a Na t seconden is de situatie zoals in de figuur.



$d = \sqrt{(200 - 20t)^2 + (80 - 12t)^2}$

$= \sqrt{40\,000 - 8000t + 400t^2 + 6400 - 1920t + 144t^2}$

$= \sqrt{544t^2 - 9920t + 46\,400}$

b Voer in $y_1 = \sqrt{544x^2 - 9920x + 46\,400}$ en

$y_2 = \text{nDeriv}(y_1, x, x)$ of $y_2 = \text{d}/\text{d}x(y_1, x)$.

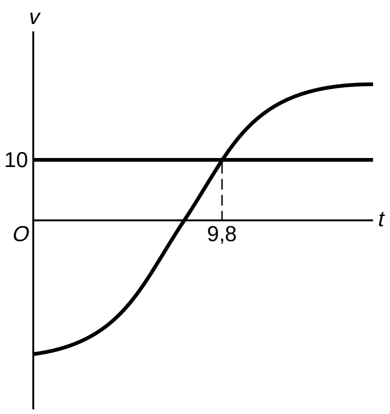
$v(4) = y_2(4) \approx -22{,}4$

De gevraagde snelheid is ongeveer 22,4 m/s.

c De optie minimum bij y_1 geeft $x \approx 9{,}1$ en $y \approx 34{,}3$.

De minimale afstand is ongeveer 34,3 m.

d Voer in $y_3 = 10$.



De optie intersect met y_2 en y_3 geeft $x \approx 9{,}8$.

Dus vanaf $t \approx 9{,}8$.