hoofdstuk

08 Diagnostische toets

bladzijde 120

1 a $6a^3 \cdot 8(a^2)^2 = 6a^3 \cdot 8a^4 = 48a^7$
b $(6a)^3 \cdot (8a^2)^2 = 216a^3 \cdot 64a^4 = 13\,824a^7$
c $\dfrac{(6a^2)^3}{(2a)^4} = \dfrac{216a^6}{16a^4} = 13\frac{1}{2}a^2$
d $(2a^2)^4 - (3a^3)^2 = 16a^8 - 9a^6$
e $(ab^2)^4 \cdot a^2b = a^4b^8 \cdot a^2b = a^6b^9$
f $\left(\dfrac{6a^2}{2a}\right)^4 = (3a)^4 = 81a^4$

2 a $a^{-2} = \dfrac{1}{a^2}$
b $10ab^{-2} = \dfrac{10a}{b^2}$
c $(4a)^{-2} \cdot 3b^{-4} = \dfrac{1}{(4a)^2} \cdot \dfrac{3}{b^4} = \dfrac{3}{16a^2b^4}$

3 a $\dfrac{1}{x^3} = x^{-3}$
b $\dfrac{1}{x^2 \cdot \sqrt{x}} = \dfrac{1}{x^2 \cdot x^{\frac{1}{2}}} = \dfrac{1}{x^{2\frac{1}{2}}} = x^{-2\frac{1}{2}}$
c $\sqrt[3]{x^2} = x^{\frac{2}{3}}$
d $x^3 \cdot \sqrt[5]{x^3} = x^3 \cdot x^{\frac{3}{5}} = x^{3+\frac{3}{5}} = x^{3\frac{3}{5}}$
e $\dfrac{x^4}{\sqrt[3]{x}} = \dfrac{x^4}{x^{\frac{1}{3}}} = x^{4-\frac{1}{3}} = x^{3\frac{2}{3}}$
f $\sqrt[3]{\dfrac{1}{x^3}} = \sqrt[3]{x^{-3}} = x^{-1}$

4 a $16\sqrt{2} = 2^4 \cdot 2^{\frac{1}{2}} = 2^{4\frac{1}{2}}$
$\sqrt[3]{32} = \sqrt[3]{2^5} = 2^{1\frac{2}{3}}$
$\sqrt[5]{\frac{1}{8}} = \sqrt[5]{2^{-3}} = 2^{-\frac{3}{5}}$
b $2^{x-4} = 2^x \cdot 2^{-4} = \dfrac{1}{2^4} \cdot 2^x = \frac{1}{16} \cdot 2^x$
$2^{x+\frac{1}{2}} = 2^x \cdot 2^{\frac{1}{2}} = \sqrt{2} \cdot 2^x$
c $2{,}16^{a-1} = 2{,}16^a \cdot 2{,}16^{-1} \approx 0{,}46 \cdot 2{,}16^a$
$1{,}27^{3a+0{,}6} = 1{,}27^{3a} \cdot 1{,}27^{0{,}6} = 1{,}27^{0{,}6} \cdot (1{,}27^3)^a \approx 1{,}15 \cdot 2{,}05^a$

5 a $5x^{1{,}2} + 6 = 20$
$5x^{1{,}2} = 14$
$x^{1{,}2} = 2{,}8$
$x = \sqrt[1{,}2]{2{,}8}$
$x \approx 2{,}358$

b $6 \cdot \sqrt[3]{x^2} + 3 = 8$
$6 \cdot \sqrt[3]{x^2} = 5$
$x^{\frac{2}{3}} = \frac{5}{6}$
$x = \sqrt[\frac{2}{3}]{\frac{5}{6}}$
$x \approx 0{,}761$

c $8x\sqrt{x} + 5 = 21$
$8x\sqrt{x} = 16$
$x^{1\frac{1}{2}} = 2$
$x = \sqrt[1\frac{1}{2}]{2}$
$x \approx 1{,}587$

6 a $K = a \cdot p^{1,3}$
Bij $p = 17$ hoort $K = 150$ $\Big\}$ $150 = a \cdot 17^{1,3}$

$\dfrac{150}{17^{1,3}} = a$

$a \approx 3,77$

b $N = a \cdot \dfrac{1}{t^{0,83}}$
Bij $t = 11$ hoort $N = 33$ $\Big\}$ $33 = a \cdot \dfrac{1}{11^{0,83}}$

$33 \cdot 11^{0,83} = a$

$a \approx 241$

7 a $y = 3^x$
$\downarrow$ 2 naar rechts
$y = 3^{x-2}$
$\downarrow$ 3 omlaag
$y = 3^{x-2} - 3$

$y = (\frac{1}{3})^x$
$\downarrow$ verm. met 4
$y = 4 \cdot (\frac{1}{3})^x$
$\downarrow$ 6 omlaag
$y = 4 \cdot (\frac{1}{3})^x - 6$

b Voer in $y_1 = 3^{x-2} - 3$.

Tabel:

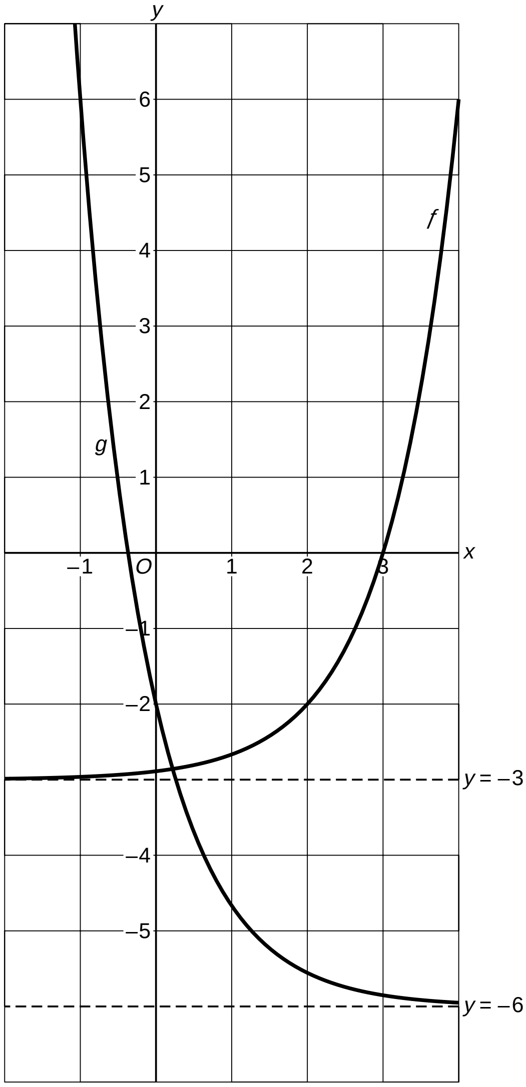
x	-1	0	1	2	3	4
$f(x)$	$-2,96$	$-2,9$	$-2,7$	-2	0	6

Voer in $y_2 = 4 \cdot \left(\frac{1}{3}\right)^x - 6$.

Tabel:

x	-1	0	1	2	3
$g(x)$	6	-2	$-4,7$	$-5,6$	$-5,9$

$B_f = \langle -3, \rightarrow \rangle$
$B_g = \langle -6, \rightarrow \rangle$



c De optie intersect geeft $x \approx 0,22$.
Aflezen $f(x) \geq g(x)$ geeft $x \geq 0,22$.

d $B_f = \langle -3, \rightarrow \rangle$, dus de vergelijking $f(x) = p$ heeft geen oplossingen voor $p \leq -3$.

8 a $5^{x-1} = 125$
$5^{x-1} = 5^3$
$x - 1 = 3$
$x = 4$

b $3^{2x-5} = \frac{1}{27}$
$3^{2x-5} = 3^{-3}$
$2x - 5 = -3$
$2x = 2$
$x = 1$

c $2 \cdot 4^{2x-1} - 3 = 61$
$2 \cdot 4^{2x-1} = 64$
$4^{2x-1} = 32$
$(2^2)^{2x-1} = 2^5$
$2^{4x-2} = 2^5$
$4x - 2 = 5$
$4x = 7$
$x = \frac{7}{4}$

bladzijde 121

9 a $7^{x-3} = 20$
$x - 3 = {}^7\log(20)$
$x = 3 + {}^7\log(20)$

b $6 \cdot 2^x + 5 = 23$
$6 \cdot 2^x = 18$
$2^x = 3$
$x = {}^2\log(3)$

c $10 \cdot (\frac{1}{2})^{2x-1} = 600$
$(\frac{1}{2})^{2x-1} = 60$
$2x - 1 = {}^{\frac{1}{2}}\log(60)$
$2x = 1 + {}^{\frac{1}{2}}\log(60)$
$x = \frac{1}{2} + \frac{1}{2} \cdot {}^{\frac{1}{2}}\log(60)$

10 a ${}^2\log(256) = {}^2\log(2^8) = 8$
b ${}^3\log(3\sqrt{3}) = {}^3\log(3^1 \cdot 3^{\frac{1}{2}}) = {}^3\log(3^{1\frac{1}{2}}) = 1\frac{1}{2}$
c ${}^5\log(\frac{1}{25}) = {}^5\log(5^{-2}) = -2$

11 a ${}^2\log(x) = -3$
$x = 2^{-3}$
$x = \frac{1}{8}$

b ${}^3\log(x - 4) = 2$
$x - 4 = 3^2$
$x - 4 = 9$
$x = 13$

c ${}^4\log(x^2 - 5) = 1$
$x^2 - 5 = 4^1$
$x^2 = 9$
$x = 3 \quad \vee \quad x = -3$

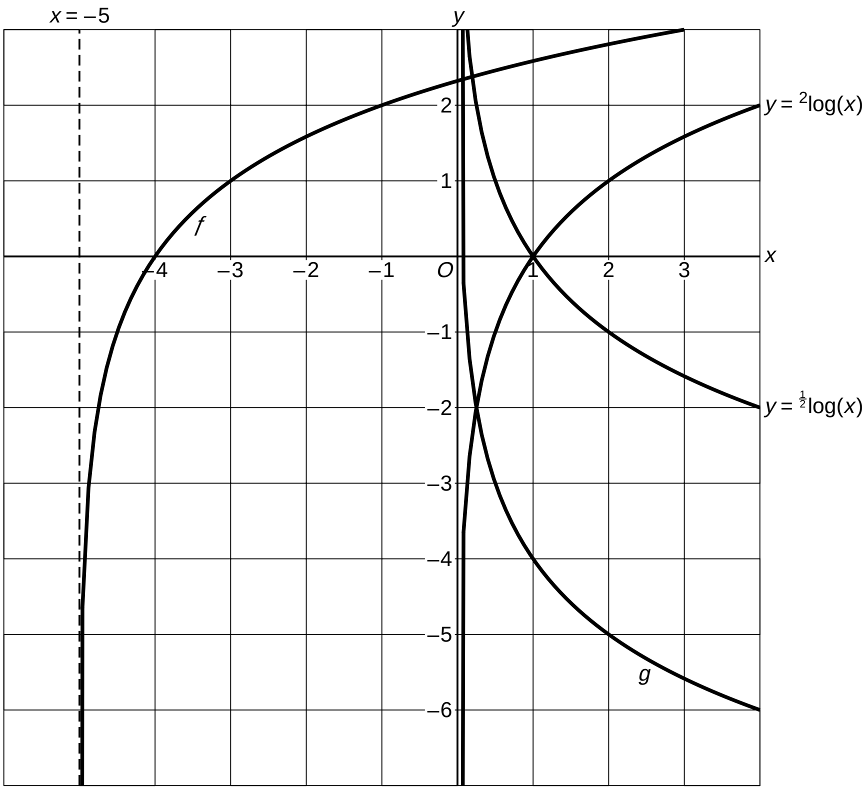
12 a $y = {}^{2}\log(x)$ $\quad$ $y = {}^{\frac{1}{2}}\log(x)$

$\downarrow$ 5 naar links $\quad$ $\downarrow$ 4 omlaag

$y = {}^{2}\log(x+5)$ $\quad$ $y = {}^{\frac{1}{2}}\log(x) - 4$

b

x	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8
${}^{2}\log(x)$	-3	-2	-1	0	1	2	3
${}^{\frac{1}{2}}\log(x)$	3	2	1	0	-1	-2	-3



$D_f = \langle -5, \rightarrow \rangle$

$D_g = \langle 0, \rightarrow \rangle$

13 a Exponentiële afname, dus $H = b \cdot g^t$.

$b = 30$ en $g = 0{,}917$, dus $H = 30 \cdot 0{,}917^t$.

b $t = 5$ geeft $H = 30 \cdot 0{,}917^5 \approx 19{,}45$.

Dus na 5 dagen is er 19,45 mg.

c Voer in $y_1 = 30 \cdot 0{,}917^x$ en $y_2 = 2$.

De optie intersect geeft $x \approx 31{,}3$.

Dus na 32 dagen.

14 a $\frac{51}{38} \approx 1{,}342 \quad \frac{68}{51} \approx 1{,}333 \quad \frac{90}{68} \approx 1{,}324 \quad \frac{120}{90} \approx 1{,}333$

De uitkomsten verschillen weinig, dus exponentiële groei.

b $N = b \cdot g^t$ met $b = 38$ en $g = 1{,}33$, dus $N = 38 \cdot 1{,}33^t$.

c Bij 1 januari 2011 hoort $t = \frac{21}{3} = 7$.

$t = 7$ geeft $N = 38 \cdot 1{,}33^7 \approx 280$.

15 a De groeifactor per 10 dagen is $\frac{1500}{2500} = 0{,}6$.

De groeifactor per dag is $0{,}6^{\frac{1}{10}} \approx 0{,}950$.
$H = b \cdot g^t$ met $g = 0{,}950$ en t in dagen, dus $H = b \cdot 0{,}950^t$.
Bij $t = 3$ hoort $H = 2500$, dus $2500 = b \cdot 0{,}950^3$

$$b = \frac{2500}{0{,}950^3}$$
$$b \approx 2914$$

Dus $H = 2914 \cdot 0{,}950^t$.
De ballon werd gevuld met 2914 cm^3 helium.

b Voer in $y_1 = 2914 \cdot 0{,}950^x$ en $y_2 = 500$.
De optie intersect geeft $x \approx 34{,}4$.
Dus na 34,4 dagen.

16 a Bij 1990 hoort $t = 5$.
$t = 5$ geeft $\log(N) = -\frac{1}{15} \cdot 5 + 3$
$\log(N) = 2\frac{2}{3}$
$N = 10^{2\frac{2}{3}} \approx 464$

b $N = 250$ geeft $-\frac{1}{15}t + 3 = \log(250)$
$-\frac{1}{15}t + 3 \approx 2{,}40$
$-\frac{1}{15}t \approx -0{,}60$
$t \approx 9{,}0$

Dus in $1985 + 9 = 1994$.

17 a $\log(W) = 0{,}6t + 2{,}4$
$W = 10^{0{,}6t+2{,}4}$
$W = 10^{0{,}6t} \cdot 10^{2{,}4}$
$W = 10^{2{,}4} \cdot (10^{0{,}6})^t$
$W \approx 251 \cdot 4{,}0^t$
Dus $W = 251 \cdot 4{,}0^t$.

b $2x + 2{,}5\log(y) = 4$
$2{,}5\log(y) = -2x + 4$
$\log(y) = -0{,}8x + 1{,}6$
$y = 10^{-0{,}8x+1{,}6}$
$y = 10^{-0{,}8x} \cdot 10^{1{,}6}$
$y = 10^{1{,}6} \cdot (10^{-0{,}8})^x$
$y \approx 39{,}8 \cdot 0{,}2^x$
Dus $y = 39{,}8 \cdot 0{,}2^x$.