hoofdstuk

10 Diagnostische toets

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1 $A(\cos 40°, \sin 40°) \approx A(0{,}77;\ 0{,}64)$
De draaiingshoek die bij het punt B hoort is $40° + 120° = 160°$.
$B(\cos 160°, \sin 160°) \approx B(-0{,}94;\ 0{,}34)$
De draaiingshoek die bij het punt C hoort is $40° - 120° = -80°$.
$C\left(\cos(-80°), \sin(-80°)\right) \approx C(0{,}17;\ -0{,}98)$

2 a $y_A = 0{,}9$, dus $\sin \alpha = 0{,}9$.
De GR geeft $\sin^{-1}(0.9) \approx 64°$.
Dus $\alpha \approx 180° - 64° = 116°$.
b $x_B = 0{,}9$, dus $\cos \beta = 0{,}9$.
De GR geeft $\cos^{-1}(0.9) \approx 26°$.
Dus $\beta \approx -26°$ en $\angle AOB \approx 116° + 26° = 142°$.

3 a $x_P = \cos 10 \text{ rad} \approx -0{,}84$ en $y_P = \sin 10 \text{ rad} \approx -0{,}54$ dus $P(-0{,}84;\ -0{,}54)$.
b De afstand is $5\frac{1}{2}\pi$, dus $P(0, -1)$.

4 a $\frac{3}{4}\pi \text{ rad} = \frac{3}{4} \cdot 180° = 135°$
b $\frac{1}{5} \text{ rad} = \frac{1}{5} \cdot 180° = 36°$
c $0{,}6 \text{ rad} = 0{,}6 \cdot \dfrac{180°}{\pi} \approx 34{,}4°$
d $26\pi \text{ rad} = 26 \cdot 180° = 4680°$
e $\frac{2}{3}\pi \text{ rad} = \frac{2}{3} \cdot 180° = 120°$
f $\frac{2}{3} \text{ rad} = \frac{2}{3} \cdot \dfrac{180°}{\pi} \approx 38{,}2°$

5 a $270° = \dfrac{270}{180}\pi \text{ rad} = 1\frac{1}{2}\pi \text{ rad}$
b $-60° = \dfrac{-60}{180}\pi \text{ rad} = -\frac{1}{3}\pi \text{ rad}$
c $150° = \dfrac{150}{180}\pi \text{ rad} = \frac{5}{6}\pi \text{ rad}$
d $330° = \dfrac{330}{180}\pi \text{ rad} = 1\frac{5}{6}\pi \text{ rad}$
e $40° = \dfrac{40}{180}\pi \text{ rad} = \frac{2}{9}\pi \text{ rad}$
f $-70° = \dfrac{-70}{180}\pi \text{ rad} = -\frac{7}{18}\pi \text{ rad}$

6 a $\alpha = \sin^{-1}(0.73) \approx 0{,}82$

b $\alpha = \cos^{-1}(\frac{6}{7}) \approx 0{,}54$

c $\alpha = \sin^{-1}(\frac{1}{2}\sqrt{2}) \approx 0{,}79$

7 a $y = \sin(x)$

$\downarrow$ verm. t.o.v. x-as, 3

$y = 3\sin(x)$

$\downarrow$ translatie $(\frac{1}{2}\pi, 2)$

$y = 2 + 3\sin(x - \frac{1}{2}\pi)$

b $y = \sin(x)$

$\downarrow$ verm. t.o.v. y-as, $\frac{1}{2}$

$y = \sin(2x)$

$\downarrow$ translatie $(\frac{1}{4}, -3)$

$y = -3 + \sin\left(2(x - \frac{1}{4})\right)$

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8 $y = \cos(x)$

$\downarrow$ translatie $(\frac{3}{4}\pi, 1)$

$y = 1 + \cos(x - \frac{3}{4}\pi)$

$\downarrow$ verm. t.o.v. y-as, 4

$y = 1 + \cos(\frac{1}{4}x - \frac{3}{4}\pi)$

$\downarrow$ verm. t.o.v. x-as, -2

$y = -2(1 + \cos(\frac{1}{4}x - \frac{3}{4}\pi)$

Dus $f(x) = -2 - 2\cos(\frac{1}{4}x - \frac{3}{4}\pi)$ ofwel $f(x) = -2 - 2\cos\left(\frac{1}{4}(x - 3\pi)\right)$.

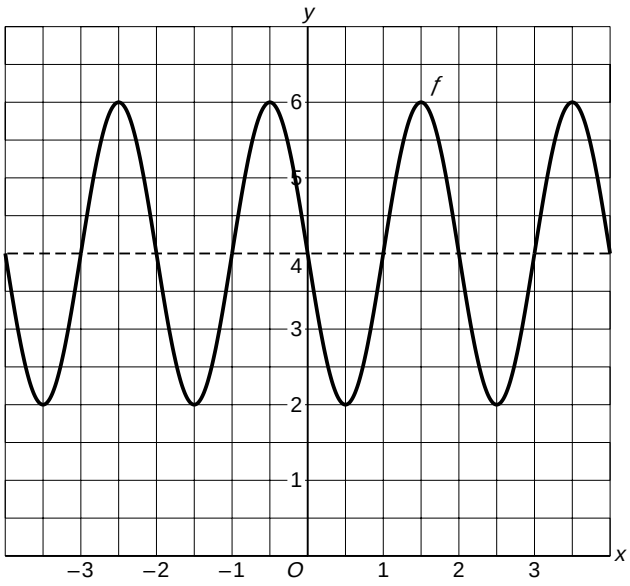
9 a $f(x) = 4 + 2\sin\left(\pi(x - 1)\right)$

evenwichtsstand 4

amplitude 2

periode $\dfrac{2\pi}{\pi} = 2$

beginpunt $(1, 4)$



b Voer in $y_1 = 4 + 2\sin\left(\pi(x - 1)\right)$ en $y_2 = 5$.

De optie intersect geeft $x \approx -2{,}83$, $x \approx -2{,}17$, $x \approx -0{,}83$, $x \approx -0{,}17$, $x \approx 1{,}17$, $x \approx 1{,}83$, $x \approx 3{,}17$ en $x \approx 3{,}83$.

Aflezen $f(x) \geq 5$ geeft

$-2{,}83 \leq x \leq -2{,}17 \ \vee \ -0{,}83 \leq x \leq -0{,}17 \ \vee \ 1{,}17 \leq x \leq 1{,}83 \ \vee \ 3{,}17 \leq x \leq 3{,}83$.

c De GR geeft $\left[\dfrac{\mathrm{d}y}{\mathrm{d}x}\right]_{x=1} \approx 6{,}28$, dus de gevraagde helling is 6,28.

10 a $f(x) = a + b\sin\big(c(x-d)\big)$

$a =$ evenwichtsstand $= \dfrac{10 + -30}{2} = -10$

$b =$ amplitude $= 10 - -10 = 20$

$c = \dfrac{2\pi}{\text{periode}} = \dfrac{2\pi}{30} = \dfrac{\pi}{15}$

$d = 10$, want beginpunt $(10, -10)$

Dus $f(x) = -10 + 20\sin\left(\dfrac{\pi}{15}(x-10)\right)$.

b $f(x) = a + b\cos\big(c(x-d)\big)$

$a = -10$, $b = 20$ en $c = \dfrac{\pi}{15}$

$d = 17{,}5$, want beginpunt $(17{,}5;\, 10)$

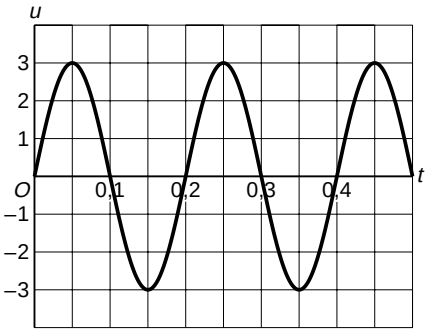
Dus $f(x) = -10 + 20\cos\left(\dfrac{\pi}{15}(x-17{,}5)\right)$.

11 a De amplitude is 3 cm.

$10\pi = \dfrac{2\pi}{\frac{1}{5}}$, dus de trillingstijd is $\frac{1}{5}$ seconde.

De frequentie is 5 hertz.

b De periode is $\frac{1}{5}$ seconde.



c 0,2 periode is $0{,}2 \cdot \frac{1}{5} = 0{,}04$ seconde

Dus voor Q geldt $u = 3\sin\big(10\pi(t-0{,}04)\big)$.

12 a De afstand AB is $\sqrt{100^2 + 21^2} \approx 102{,}18$ m.

De omtrek van het vat is: $2\pi \cdot 0{,}25 \approx 1{,}571$ m.

Aantal omwentelingen is $\dfrac{102{,}18}{1{,}571} \approx 65$.

b Lees af: 1 omwenteling duurt 2 seconden.

Dus na $62 \cdot 2 = 130$ seconden is het vat in B.

c Er wordt 102,18 m afgelegd in 130 seconden,

dus de snelheid is $\dfrac{102{,}18}{130} = 0{,}786$ m/s $\approx 2{,}83$ km/uur.