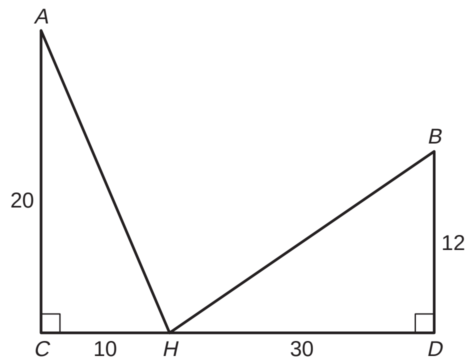
hoofdstuk

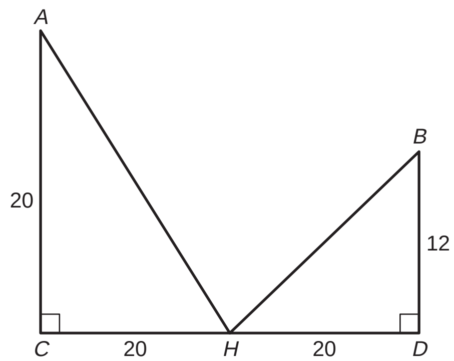
# 05 Vergelijkingen en ongelijkheden

bladzijde 148

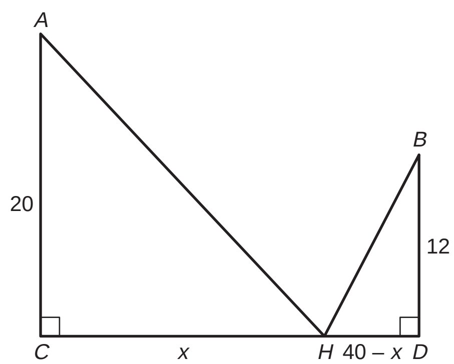
- $AH^2 = 20^2 + 10^2 = 500$
  $AH = \sqrt{500} \approx 22{,}4$ km
  $BH^2 = 12^2 + 30^2 = 1044$
  $BH = \sqrt{1044} \approx 32{,}3$ km




- $AH^2 = 20^2 + 20^2 = 800$
  $AH = \sqrt{800} \approx 28{,}3$ km
  $BH^2 = 12^2 + 20^2 = 544$
  $BH = \sqrt{544} \approx 23{,}3$ km




- $AH^2 = x^2 + 20^2 = x^2 + 400$
  $AH = \sqrt{x^2 + 400}$ km
- $DH = CD - CH = 40 - x$
  $BH^2 = 12^2 + (40 - x)^2 = 144 + (40 - x)^2$
  $BH = \sqrt{144 + (40 - x)^2}$
- Je moet de vergelijking
  $\sqrt{x^2 + 400} = \sqrt{144 + (40 - x)^2}$ oplossen.
  Voer in $y_1 = \sqrt{x^2 + 400}$ en
  $y_2 = \sqrt{144 + (40 - x))^2}$.
  De optie intersect geeft $x = 16{,}8$.
  Dus $x = 16{,}8$ km.
- $AH + BH = \sqrt{x^2 + 400} + \sqrt{144 + (40 - x)^2}$
  Voer in $y_1 = \sqrt{x^2 + 400} + \sqrt{144 + (40 - x)^2}$.
  De optie minimum geeft $x = 25$ en $y \approx 51{,}2$.
  Dus $AH + BH$ is niet minimaal als $AH = BH$, want $25 \neq 16{,}8$.



# 5.1 Stelsels vergelijkingen

bladzijde 150

**1** a Tien broden kosten 16 euro.
Blijft over $60 - 16 = 44$ euro.
Dus kan hij nog $\frac{44}{0{,}40} = 110$ bolletjes kopen.

b 90 bolletjes kosten $90 \times 0{,}4 = 36$ euro.
Blijft over $60 - 36 = 24$ euro.
Dus kan hij nog $\frac{24}{1{,}60} = 15$ broden kopen.

c $1{,}6x + 0{,}4y = 60$.

bladzijde 151

**2** a $15x + 12y = 2520$
$12y = -15x + 2520$
$y = -1{,}25x + 210$

b $3p - 2q = 16\frac{1}{2}$
$3p = 2q + 16\frac{1}{2}$
$p = \frac{2}{3}q + 5\frac{1}{2}$

c $5a - 2b = 16$
$-2b = -5a + 16$
$b = 2\frac{1}{2}a - 8$

**3** a $3x - y = 6$

| $x$ | 0 | 2 |
|---|---|---|
| $y$ | $-6$ | 0 |

$x + y = 1$

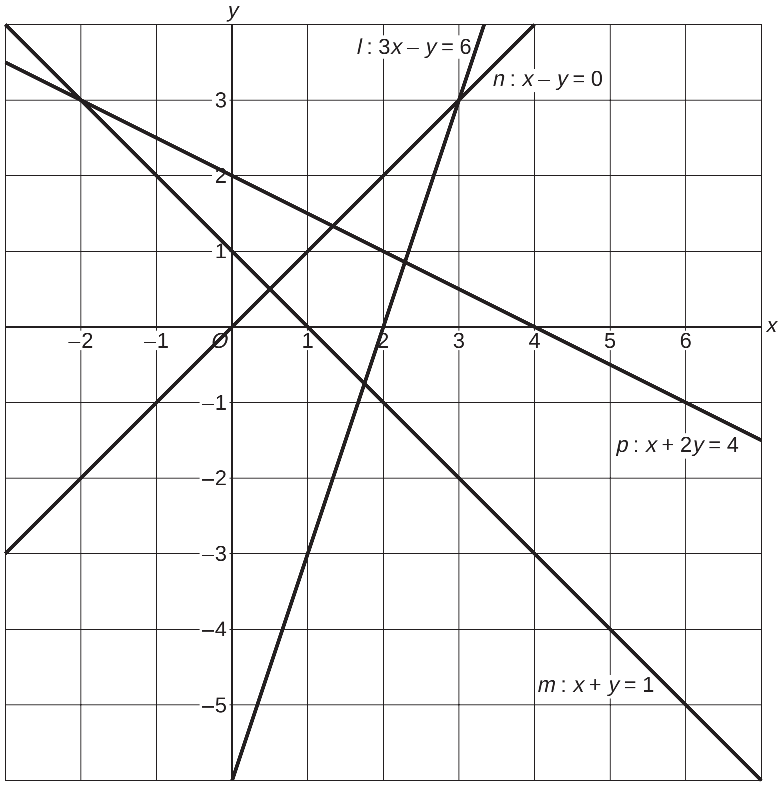
| $x$ | 0 | 1 |
|---|---|---|
| $y$ | 1 | 0 |

$x - y = 0$

| $x$ | 0 | 1 |
|---|---|---|
| $y$ | 0 | 1 |

$x + 2y = 4$

| $x$ | 0 | 4 |
|---|---|---|
| $y$ | 2 | 0 |




b $l$: $3x - y = 6$
$-y = -3x + 6$
$y = 3x - 6$
$\text{rc}_l = 3$

$m$: $x + y = 1$
$y = -x + 1$
$\text{rc}_m = -1$

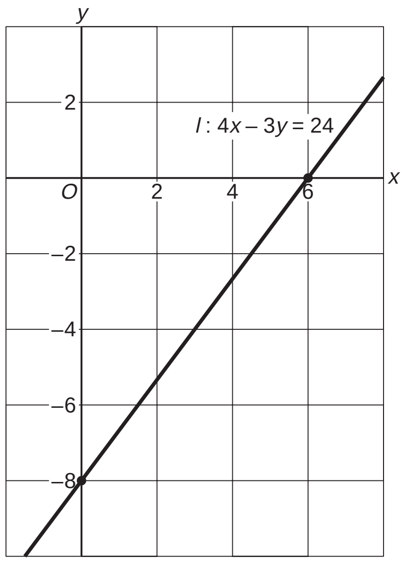
$n$: $x - y = 0$
$-y = -x$
$y = x$
$\text{rc}_n = 1$

$p$: $x + 2y = 4$
$2y = -x + 4$
$y = -\frac{1}{2}x + 2$
$\text{rc}_p = -\frac{1}{2}$

**4** a $l: 4x - 3y = 24$

Snijden met de $x$-as dus
$y = 0$
$4x = 24$
$x = 6$
dus $(6, 0)$

Snijden met de $y$-as dus
$x = 0$
$-3y = 24$
$y = -8$
dus $(0, -8)$




b $A(8, 3)$ invullen geeft $4 \cdot 8 - 3 \cdot 3 = 24$
$32 - 9 = 24$
Klopt niet, dus $A$ ligt niet op $l$.
$B(18, 16)$ invullen geeft $4 \cdot 18 - 3 \cdot 16 = 24$
$72 - 48 = 24$
Klopt dus $B$ ligt op $l$.
$C(-30, -48)$ invullen geeft $4 \cdot -30 - 3 \cdot -48 = 24$
$-120 + 144 = 24$
Klopt dus $C$ ligt op $l$.

c $(16, p)$ invullen geeft $4 \cdot 16 - 3 \cdot p = 24$
$64 - 3p = 24$
$-3p = -40$
$p = 13\frac{1}{3}$

d $(q, 48)$ invullen geeft $4 \cdot q - 3 \cdot 48 = 24$
$4q - 144 = 24$
$4q = 168$
$q = 42$

bladzijde 152

**5** a $l: 3x - 4y = 7$
$-4y = -3x + 7$
$y = \frac{3}{4}x - \frac{7}{4}$
$\text{rc}_l = \frac{3}{4}$

$m: 3x - 4y = -8$
$-4y = -3x - 8$
$y = \frac{3}{4}x + 2$
$\text{rc}_m = \frac{3}{4}$

b Ze hebben dezelfde richtingscoëfficiënt, namelijk $\frac{3}{4}$.

c $A(5, 1)$ invullen in $3x - 4y = c$ geeft $3 \cdot 5 - 4 \cdot 1 = c$
$15 - 4 = c$
$c = 11$

d $B(3, -1)$ invullen in $3x - 4y = c$ geeft $3 \cdot 3 - 4 \cdot -1 = c$
$9 + 4 = c$
$c = 13$
dus $k: 3x - 4y = 13$

**6** $A(5, 8)$ invullen in $2x + y = c$ geeft $2 \cdot 5 + 8 = c$
$10 + 8 = c$
$c = 18$
dus $m: 2x + y = 18$

**7** $4x + 7y = 5{,}8$

**8** Stel $x$ kaartjes van 10 euro en $y$ kaartjes van 15 euro dan geldt $10x + 15y = 4300$.

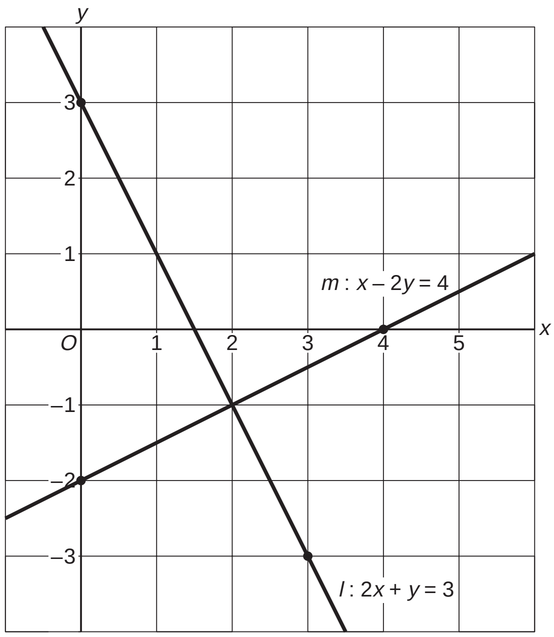
**9** Als hij 50 munten van 2 euro heeft, dan is de waarde 100 euro, 13 euro teveel.
Hij heeft dus 13 munten van 1 euro en 37 munten van 2 euro.

**10** a $2x + y = 3$

| $x$ | 0 | 3 |
|---|---|---|
| $y$ | 3 | $-3$ |

$x - 2y = 4$

| $x$ | 0 | 4 |
|---|---|---|
| $y$ | $-2$ | 0 |




b $(2, -1)$ is het snijpunt.

c $(2, -1)$ is oplossing van $x - 2y = 4$ en van $2x + y = 3$.

bladzijde 154

**11** a

$$\begin{cases} 5x - 4y = -8 \\ -x + 4y = -12 \end{cases} \quad +$$

$$4x = -20$$

$$\left.\begin{array}{l} x = -5 \\ -x + 4y = -12 \end{array}\right\} \quad \begin{array}{l} 5 + 4y = -12 \\ 4y = -17 \\ y = -4\frac{1}{4} \end{array}$$

De oplossing is $(-5, -4\frac{1}{4})$.

b

$$\begin{cases} -2x + y = 7 \\ -2x + 3y = -1 \end{cases} \quad -$$

$$-2y = 8$$

$$\left.\begin{array}{l} y = -4 \\ -2x + y = 7 \end{array}\right\} \quad \begin{array}{l} -2x - 4 = 7 \\ -2x = 11 \\ x = -5\frac{1}{2} \end{array}$$

De oplossing is $(-5\frac{1}{2}, -4)$.

c

$$\begin{cases} -x - 3y = -8 \\ -2x + 3y = -1 \end{cases} \quad +$$

$$-3x = -9$$

$$\left.\begin{array}{l} x = 3 \\ -2x + 3y = -1 \end{array}\right\} \quad \begin{array}{l} -6 + 3y = -1 \\ 3y = 5 \\ y = \frac{5}{3} = 1\frac{2}{3} \end{array}$$

De oplossing is $(3, 1\frac{2}{3})$.

**12** a $\begin{cases} 3x - 4y = 7 \\ 2x + 3y = 16 \end{cases}$ +

$5x - y = 23$

Nee, er is geen variabele geëlimineerd.

b $\begin{cases} 3x - 4y = 7 \\ 2x + 3y = 16 \end{cases}$ −

$x - 7y = -9$

Nee, er is geen variabele geëlimineerd.

bladzijde 155

**13** a $\begin{cases} 3x + 5y = -7 \\ 2x + y = 0 \end{cases} \left| \begin{matrix} 1 \\ 5 \end{matrix} \right|$ geeft $\begin{cases} 3x + 5y = -7 \\ 10x + 5y = 0 \end{cases}$ −

$-7x = -7$

$\left. \begin{matrix} x = 1 \\ 2x + y = 0 \end{matrix} \right\}$ $2 + y = 0$

$y = -2$

De oplossing is $(1, -2)$.

b $\begin{cases} 2x - 4y = 6 \\ 3x - y = 19 \end{cases} \left| \begin{matrix} 1 \\ 4 \end{matrix} \right|$ geeft $\begin{cases} 2x - 4y = 6 \\ 12x - 4y = 76 \end{cases}$ −

$-10x = -70$

$\left. \begin{matrix} x = 7 \\ 2x - 4y = 6 \end{matrix} \right\}$ $14 - 4y = 6$

$-4y = -8$

$y = 2$

De oplossing is $(7, 2)$.

c $\begin{cases} 4x + y = 13 \\ x - 2y = 1 \end{cases} \left| \begin{matrix} 2 \\ 1 \end{matrix} \right|$ geeft $\begin{cases} 8x + 2y = 26 \\ x - 2y = 1 \end{cases}$ +

$9x = 27$

$\left. \begin{matrix} x = 3 \\ 4x + y = 13 \end{matrix} \right\}$ $12 + y = 13$

$y = 1$

De oplossing is $(3, 1)$.

**14** a $\begin{cases} 5x + 2y = 69 \\ x + 3y = -7 \end{cases} \left|\begin{matrix} 1 \\ 5 \end{matrix}\right|$ geeft $\begin{cases} 5x + 2y = 69 \\ 5x + 15y = -35 \end{cases}$ $-$

$-13y = 104$

$\left.\begin{matrix} y = -8 \\ x + 3y = -7 \end{matrix}\right\}$ $x - 24 = -7$

$x = 17$

De oplossing is $(17, -8)$.

b $\begin{cases} 2x - 5y = -19 \\ 5x + 4y = 35 \end{cases} \left|\begin{matrix} 4 \\ 5 \end{matrix}\right|$ geeft $\begin{cases} 8x - 20y = -76 \\ 25x + 20y = 175 \end{cases}$ $+$

$33x = 99$

$\left.\begin{matrix} x = 3 \\ 2x - 5y = -19 \end{matrix}\right\}$ $6 - 5y = -19$

$-5y = -25$

$y = 5$

De oplossing is $(3, 5)$.

c $\begin{cases} 0{,}8x + 0{,}2y = 1 \\ 0{,}3x - 0{,}3y = 1{,}5 \end{cases} \left|\begin{matrix} 3 \\ 2 \end{matrix}\right|$ geeft $\begin{cases} 2{,}4x + 0{,}6y = 3 \\ 0{,}6x - 0{,}6y = 3 \end{cases}$ $+$

$3x = 6$

$\left.\begin{matrix} x = 2 \\ 0{,}8x + 0{,}2y = 1 \end{matrix}\right\}$ $1{,}6 + 0{,}2y = 1$

$0{,}2y = -0{,}6$

$y = -3$

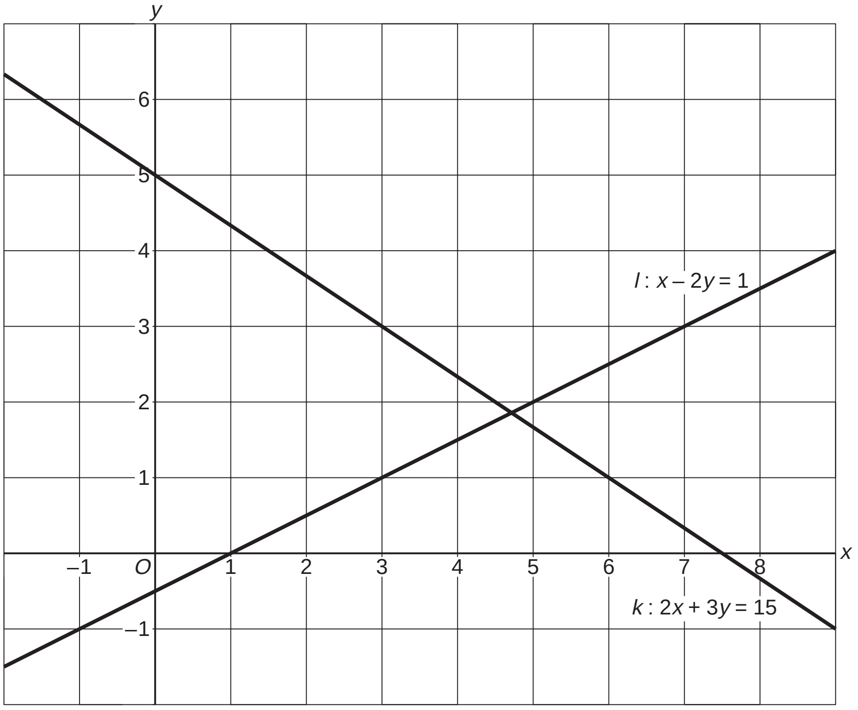
De oplossing is $(2, -3)$.

**15** a $2x + 3y = 15$

| $x$ | $0$ | $7\frac{1}{2}$ |
|---|---|---|
| $y$ | $5$ | $0$ |

$x - 2y = 1$

| $x$ | $-1$ | $1$ |
|---|---|---|
| $y$ | $-1$ | $0$ |




b Je moet het stelsel $\begin{cases} 2x + 3y = 15 \\ x - 2y = 1 \end{cases}$ oplossen.

$$\begin{cases} 2x + 3y = 15 \\ x - 2y = 1 \end{cases} \left| \begin{matrix} 2 \\ 3 \end{matrix} \right| \text{ geeft } \begin{cases} 4x + 6y = 30 \\ 3x - 6y = 3 \end{cases}$$

$$\underline{\phantom{7x = 33}} \; +$$

$$7x \quad = 33$$

$$\left. \begin{matrix} x = \frac{33}{7} = 4\frac{5}{7} \\ 2x + 3y = 15 \end{matrix} \right\} \begin{matrix} 9\frac{3}{7} + 3y = 15 \\ 3y = 5\frac{4}{7} \\ y = 1\frac{6}{7} \end{matrix}$$

Het snijpunt is $(4\frac{5}{7}, 1\frac{6}{7})$.

**16**

$$\begin{cases} 3x - 2y = -12 \\ x + 4y = 38 \end{cases} \left| \begin{matrix} 2 \\ 1 \end{matrix} \right| \text{ geeft } \begin{cases} 6x - 4y = -24 \\ x + 4y = 38 \end{cases}$$

$$\underline{\phantom{7x = 14}} \; +$$

$$7x \quad = 14$$

$$\left. \begin{matrix} x = 2 \\ x + 4y = 38 \end{matrix} \right\} \begin{matrix} 2 + 4y = 38 \\ 4y = 36 \\ y = 9 \end{matrix}$$

Het snijpunt is $(2, 9)$.

**17** $\begin{cases} 2x + 3y = 12 \\ y = 4x - 10 \end{cases}$ ofwel

$$\begin{cases} 2x + 3y = 12 \\ -4x + y = -10 \end{cases} \begin{vmatrix} 1 \\ 3 \end{vmatrix} \text{ geeft } \begin{cases} 2x + 3y = 12 \\ -12x + 3y = -30 \end{cases}$$

$$\begin{aligned} 14x &= 42 \;\; - \\ x &= 3 \\ y &= 4x - 10 \end{aligned} \Bigg\} \; y = 4 \cdot 3 - 10 = 2$$

Het snijpunt is $(3, 2)$.

**18** a $\begin{cases} 2x + 2y = 9 \\ y = 4x - 3 \end{cases}$ substitueren geeft

$$\begin{aligned} 2x + 2(4x - 3) &= 9 \\ 2x + 8x - 6 &= 9 \\ 10x &= 15 \\ x &= 1\tfrac{1}{2} \\ y &= 4x - 3 \end{aligned} \Bigg\} \; y = 4 \cdot 1\tfrac{1}{2} - 3 = 3$$

De oplossing is $(1\frac{1}{2}, 3)$.

b $\begin{cases} y = \frac{1}{2}x + 1 \\ 3x + 6y = 8 \end{cases}$ substitueren geeft

$$\begin{aligned} 3x + 6(\tfrac{1}{2}x + 1) &= 8 \\ 3x + 3x + 6 &= 8 \\ 6x &= 2 \\ x &= \tfrac{1}{3} \\ y &= \tfrac{1}{2}x + 1 \end{aligned} \Bigg\} \; y = \tfrac{1}{2} \cdot \tfrac{1}{3} + 1 = 1\tfrac{1}{6}$$

De oplossing is $(\frac{1}{3}, 1\frac{1}{6})$.

c $\begin{cases} x = 5y - 3 \\ 3x + 4y = 29 \end{cases}$ substitueren geeft

$$\begin{aligned} 3(5y - 3) + 4y &= 29 \\ 15y - 9 + 4y &= 29 \\ 19y &= 38 \\ y &= 2 \\ x &= 5y - 3 \end{aligned} \Bigg\} \; x = 5 \cdot 2 - 3 = 7$$

De oplossing is $(7, 2)$.

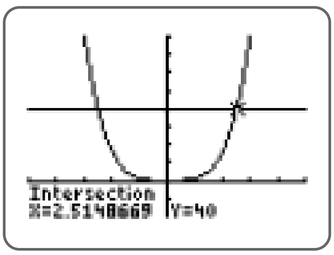
## 5.2 Hogeregraads vergelijkingen

bladzijde 158

**19** a Voer in $y_1 = x^4$.

b Voer in $y_2 = 40$.
Er zijn twee snijpunten, de vergelijking heeft dus twee oplossingen.
De optie intersect geeft $x \approx -2{,}51 \;\vee\; x \approx 2{,}51$.

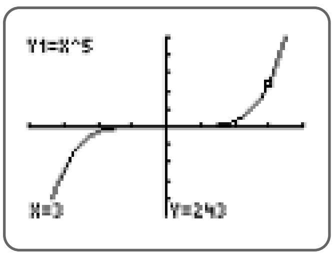
c $x^4 = -40$ heeft geen oplossingen want $x^4$ is altijd groter of gelijk aan 0.




**20** a Voer in $y_1 = x^5$.

b Voer in $y_2 = 250$.
De vergelijking $x^5 = 250$ heeft één oplossing want $y_1$ en $y_2$ hebben één snijpunt.

c Voer in $y_3 = -250$.
De vergelijking $x^5 = -250$ heeft één oplossing want $y_1$ en $y_3$ hebben één snijpunt.



**21**

a $x^6 = 20$
$x = \sqrt[6]{20} \;\vee\; x = -\sqrt[6]{20}$

b $5x^3 = 100$
$x^3 = 20$
$x = \sqrt[3]{20}$

c $x^2 + 7 = 18$
$x^2 = 11$
$x = \sqrt{11} \;\vee\; x = -\sqrt{11}$

d $3x^7 + 25 = 4$
$3x^7 = -21$
$x^7 = -7$
$x = \sqrt[7]{-7}$

e $\frac{1}{2}x^6 + 12 = 9$
$\frac{1}{2}x^6 = -3$
$x^6 = -6$
geen oplossing

f $0{,}3x^8 + 5 = 11$
$0{,}3x^8 = 6$
$x^8 = 20$
$x = \sqrt[8]{20} \;\vee\; x = -\sqrt[8]{20}$

**22**

a $3x^5 + 10 = 16$
$3x^5 = 6$
$x^5 = 2$
$x = \sqrt[5]{2}$
$x \approx 1{,}15$

b $2x^5 + 9 = 1$
$2x^5 = -8$
$x^5 = -4$
$x = \sqrt[5]{-4}$
$x \approx -1{,}32$

c $3x^4 - 5 = 10$
$3x^4 = 15$
$x^4 = 5$
$x = -\sqrt[4]{5} \;\vee\; x = \sqrt[4]{5}$
$x \approx -1{,}50 \;\vee\; x \approx 1{,}50$

d $3x^4 + 10 = 4$
$3x^4 = -6$
$x^4 = -2$
geen oplossing

e $\frac{1}{3}x^6 + 2 = 6$
$\frac{1}{3}x^6 = 4$
$x^6 = 12$
$x = -\sqrt[6]{12} \;\vee\; x = \sqrt[6]{12}$
$x \approx -1{,}51 \;\vee\; x \approx 1{,}51$

f $-\frac{1}{2}x^6 + 6 = 2$
$-\frac{1}{2}x^6 = -4$
$x^6 = 8$
$x = -\sqrt[6]{8} \;\vee\; x = \sqrt[6]{8}$
$x \approx -1{,}41 \;\vee\; x \approx 1{,}41$

**23**

a $4^3 = 64$ dus $\sqrt[3]{64} = 4$

b $x = \sqrt[3]{125} = 5$

c

| $x$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| $x^2$ | 1 | 4 | 9 | 16 | 25 | 36 | 49 | 64 | 81 |
| $x^3$ | 1 | 8 | 27 | 64 | 125 | 216 | 343 | × | × |
| $x^4$ | 1 | 16 | 81 | 256 | 625 | × | × | × | × |
| $x^5$ | 1 | 32 | 243 | 1024 | × | × | × | × | × |
| $x^6$ | 1 | 64 | 729 | × | × | × | × | × | × |

bladzijde 161

**24**

a $0{,}5x^3 - 8 = 100$
$0{,}5x^3 = 108$
$x^3 = 216$
$x = 6$

b $\frac{1}{9}x^6 - 1 = 80$
$\frac{1}{9}x^6 = 81$
$x^6 = 729$
$x = 3 \ \vee \ x = -3$

c $82 - \frac{1}{3}x^5 = 1$
$-\frac{1}{3}x^5 = -81$
$x^5 = 243$
$x = 3$

d $3(2x - 1)^2 = 147$
$(2x - 1)^2 = 49$
$2x - 1 = 7 \ \vee \ 2x - 1 = -7$
$2x = 8 \ \vee \ 2x = -6$
$x = 4 \ \vee \ x = -3$

e $5(x + 2)^3 - 36 = 99$
$5(x + 2)^3 = 135$
$(x + 2)^3 = 27$
$x + 2 = 3$
$x = 1$

f $0{,}2(4x + 1)^4 - 25 = 100$
$0{,}2(4x + 1)^4 = 125$
$(4x + 1)^4 = 625$
$4x + 1 = 5 \ \vee \ 4x + 1 = -5$
$4x = 4 \ \vee \ 4x = -6$
$x = 1 \ \vee \ x = -1\frac{1}{2}$

bladzijde 162

**25**

a opp $= 4 \cdot 3 = 12$ cm$^2$
b opp $= 4 \cdot 3 \cdot 2 + 4 \cdot 2 \cdot 2 + 2 \cdot 3 \cdot 2 = 24 + 16 + 12 = 52$ cm$^2$
c inhoud $= 4 \cdot 2 \cdot 3 = 24$ cm$^3$
d 20 bij 10 bij 15 cm
e opp $= 20 \cdot 15 = 300$ cm$^2$
f De oppervlakte is met $\frac{300}{12} = 25$ vermenigvuldigd. Dit is precies $k^2$.
g De totale oppervlakte is $20 \cdot 15 \cdot 2 + 20 \cdot 10 \cdot 2 + 10 \cdot 3 \cdot 2 = 1300$ cm$^2$.
De oppervlakte is met $\frac{1300}{52} = 25$ vermenigvuldigd.
Dus het verband bestaat ook voor de totale oppervlakte.
h Inhoud is $20 \cdot 10 \cdot 15 = 3000$ cm$^3$.
$\frac{\text{Inhoud (balk II)}}{\text{Inhoud (balk I)}} = \frac{3000}{24} = 125.$
Dit is precies $k^3$.

bladzijde 163

**26** tennisbal $\xrightarrow{\times k}$ basketbal
dus $k^2 = 20$
$k = \sqrt{20}$
Voor de inhoud is de factor $k^3 = (\sqrt{20})^3 \approx 89{,}4$.
De inhoud is ongeveer 89 keer zo groot.

**27** kleine ballon $\xrightarrow{\times k}$ grote ballon
dus $k = 2$
$k^3 = 8$
De inhoud wordt $8 \cdot 3 = 24$ liter.
De inhoud neemt toe met $24 - 3 = 21$ liter.

**28** kleine bol $\xrightarrow{\times k}$ grote bol
dus $k^3 = 3$
$k = \sqrt[3]{3}$
Voor de oppervlakte geldt de factor $k^2 = (\sqrt[3]{3})^2 \approx 2{,}1$.
Harm moet de bollen 2,1 keer zo lang bakken.

# **5.3** Ongelijkheden oplossen

bladzijde 165

TOETS VOORKENNIS

a $2x - 5 < 3x + 2$
$-x < 7$
$x > -7$

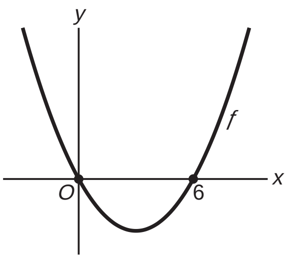
b $4x + 3 > 2x - 5$
$2x > -8$
$x > -4$

c $3(x - 1) < 6x - 1$
$3x - 3 < 6x - 1$
$-3x < 2$
$x > -\frac{2}{3}$

d $x - 6 > 3(x - 2)$
$x - 6 > 3x - 6$
$-2x > 0$
$x < 0$

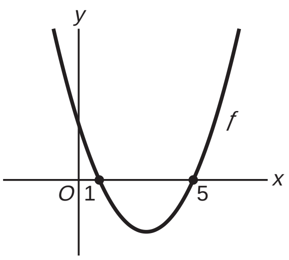
TOETS VOORKENNIS

a $x^2 - 6x > 0$
Stel $f(x) = x^2 - 6x$
$f(x) = 0$ geeft $x^2 - 6x = 0$
$x(x - 6) = 0$
$x = 0 \vee x = 6$




$f(x) > 0$ geeft $x < 0 \vee x > 6$

b $x^2 - 6x + 5 < 0$
Stel $f(x) = x^2 - 6x + 5$
$f(x) = 0$ geeft $x^2 - 6x + 5 = 0$
$(x - 1)(x - 5) = 0$
$x = 1 \vee x = 5$



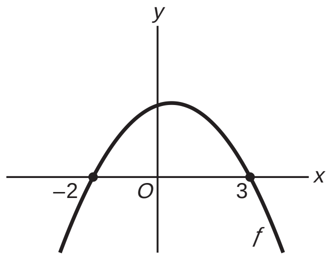

$f(x) < 0$ geeft $1 < x < 5$

c $-x^2 + x + 6 > 0$

Stel $f(x) = -x^2 + x + 6$

$f(x) = 0$ geeft

$-x^2 + x + 6 = 0$
$x^2 - x - 6 = 0$
$(x - 3)(x + 2) = 0$
$x = 3 \quad \vee \quad x = -2$



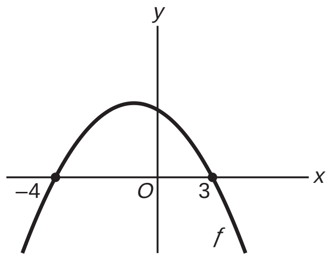

$f(x) > 0$ geeft $-2 < x < 3$

d $-x^2 - x + 12 < 0$

Stel $f(x) = -x^2 - x + 12$

$f(x) = 0$ geeft

$-x^2 - x + 12 = 0$
$x^2 + x - 12 = 0$
$(x + 4)(x - 3) = 0$
$x = -4 \quad \vee \quad x = 3$



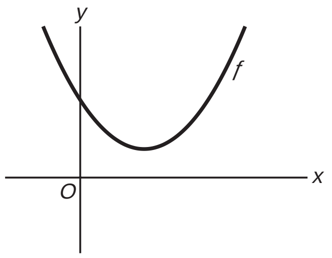

$f(x) < 0$ geeft $x < -4 \quad \vee \quad x > 3$

e $x^2 - x + 1 > 0$

Stel $f(x) = x^2 - x + 1$

$f(x) = 0$ geeft

$x^2 - x + 1 = 0$
$D = (-1)^2 - 4 \cdot 1 \cdot 1 = -3 < 0$
$f(x) = 0$ heeft geen oplossing



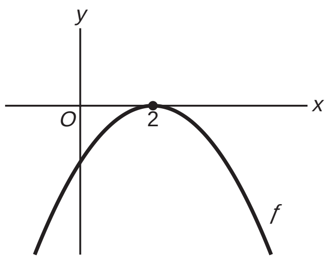

$f(x) > 0$ geeft elke $x$ is oplossing

f $-x^2 + 4x - 4 < 0$

Stel $f(x) = -x^2 + 4x - 4$

$f(x) = 0$ geeft

$-x^2 + 4x - 4 = 0$
$x^2 - 4x + 4 = 0$
$(x - 2)^2 = 0$
$x = 2$




$f(x) < 0$ geeft $x < 2 \quad \vee \quad x > 2$

**29** a Je moet aan beide kanten $5x$ aftrekken.

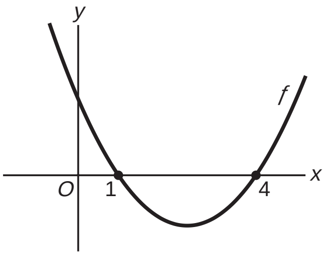
b $x^2 - 5x + 4 < 0$

Stel $f(x) = x^2 - 5x + 4$

$f(x) = 0$ geeft $x^2 - 5x + 4 = 0$

$(x - 1)(x - 4) = 0$

$x = 1 \vee x = 4$




$f(x) < 0$ geeft $1 < x < 4$

c $x^2 + 4 < 5x$ heeft dezelfde oplossingen als $x^2 - 5x + 4 < 0$, dus ook $1 < x < 4$.

bladzijde 166

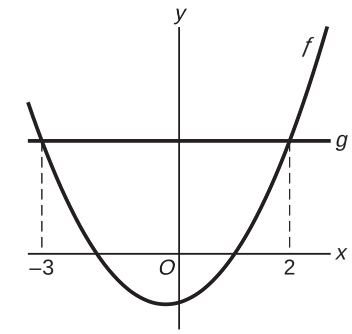
**30** a $\underbrace{x^2 + 4}_{f(x)} \geq \underbrace{6}_{g(x)}$

$x^2 + x = 6$

$x^2 + x - 6 = 0$

$(x + 3)(x - 2) = 0$

$x = -3 \vee x = 2$




Aflezen geeft $x \leq -3 \vee x \geq 2$.

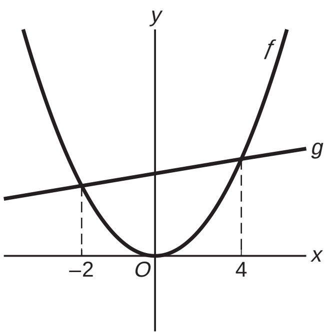
b $\underbrace{x^2}_{f(x)} \leq \underbrace{2x + 8}_{g(x)}$

$x^2 = 2x + 8$

$x^2 - 2x - 8 = 0$

$(x - 4)(x + 2) = 0$

$x = 4 \vee x = -2$




Aflezen geeft $-2 \leq x \leq 4$.

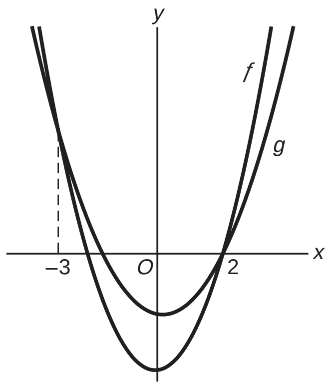
c $\underbrace{2x^2 + x - 10}_{f(x)} \geq \underbrace{x^2 - 4}_{g(x)}$

$2x^2 + x - 10 = x^2 - 4$

$x^2 + x - 6 = 0$

$(x + 3)(x - 2) = 0$

$x = -3 \vee x = 2$




Aflezen geeft $x \leq -3 \vee x \geq 2$.

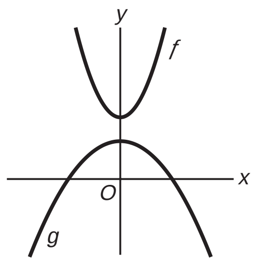
d $\underbrace{2x^2 + 7}_{f(x)} \leq \underbrace{3 - x^2}_{g(x)}$

$2x^2 + 7 = 3 - x^2$

$3x^2 = -4$

$x^2 = -\frac{4}{3}$

geen oplossing

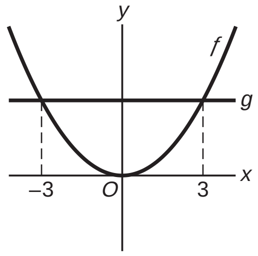



Aflezen geeft geen oplossingen.

**31** a $\underbrace{x^4}_{f(x)} \geq \underbrace{81}_{g(x)}$

$x^4 = 81$

$x = 3 \quad \vee \quad x = -3$




Aflezen geeft $x \leq -3 \quad \vee \quad x \geq 3$.

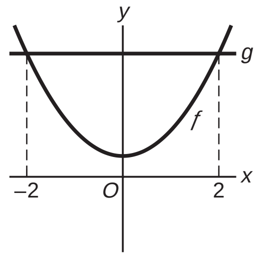
c $\underbrace{\tfrac{1}{2}x^4 + 1}_{f(x)} \leq \underbrace{9}_{g(x)}$

$\tfrac{1}{2}x^4 + 1 = 9$

$\tfrac{1}{2}x^4 = 8$

$x^4 = 16$

$x = 2 \quad \vee \quad x = -2$

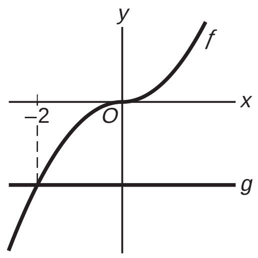



Aflezen geeft $-2 \leq x \leq 2$.

b $\underbrace{x^3}_{f(x)} \leq \underbrace{-8}_{g(x)}$

$x^3 = -8$

$x^3 = -8$




Aflezen geeft $x \leq -2$.

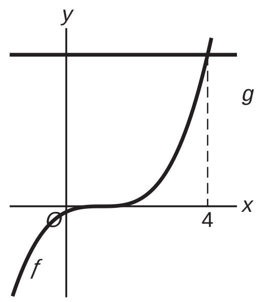
d $\underbrace{\tfrac{1}{3}(x-1)^3}_{f(x)} \geq \underbrace{9}_{g(x)}$

$\tfrac{1}{3}(x-1)^3 = 9$

$(x-1)^3 = 27$

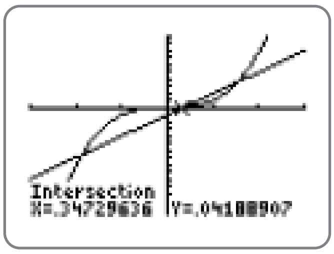
$x - 1 = 3$

$x = 4$

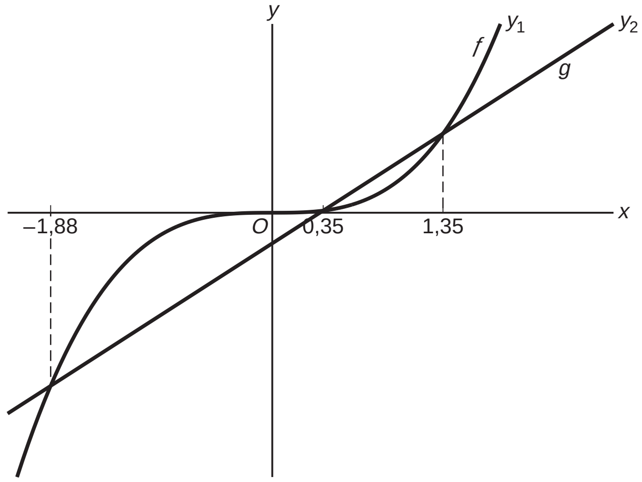



Aflezen geeft $x \geq 4$.

**32** a Voer in $y_1 = x^3$ en $y_2 = 3x - 1$.
De optie intersect geeft
$x \approx -1{,}88 \quad \vee \quad x \approx 0{,}35 \quad \vee \quad x \approx 1{,}53$.



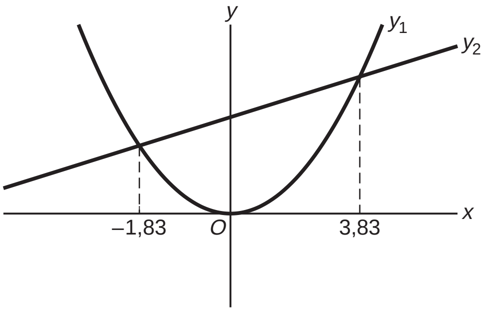

b




Aflezen geeft $x < -1{,}88 \quad \vee \quad 0{,}35 < x < 1{,}53$.

**33** a $x^2 < 2x + 7$

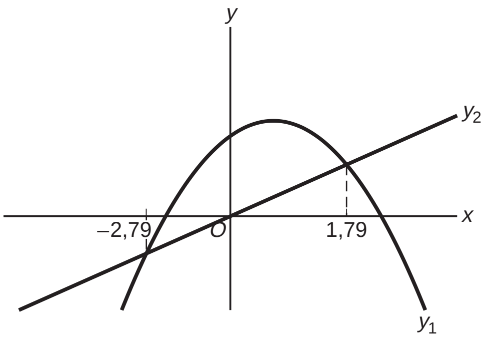
Voer in $y_1 = x^2$ en $y_2 = 2x + 7$.
De optie intersect geeft
$x \approx -1{,}83 \quad \vee \quad x \approx 3{,}83$.




Aflezen geeft $-1{,}83 < x < 3{,}83$.

b $5 - x^2 < x$

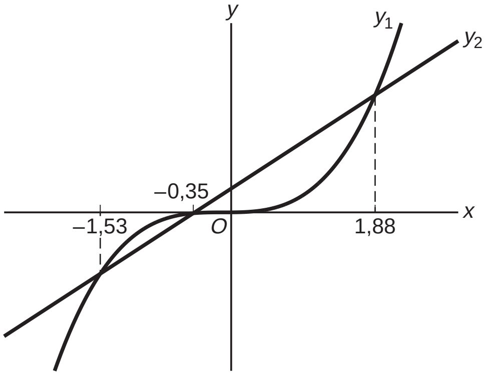
Voer in $y_1 = 5 - x^2$ en $y_2 = x$.
De optie intersect geeft
$x \approx -2{,}79 \quad \vee \quad x \approx 1{,}79$.




Aflezen geeft $x < -2{,}79 \quad \vee \quad x > 1{,}79$.

c $x^3 > 3x + 1$

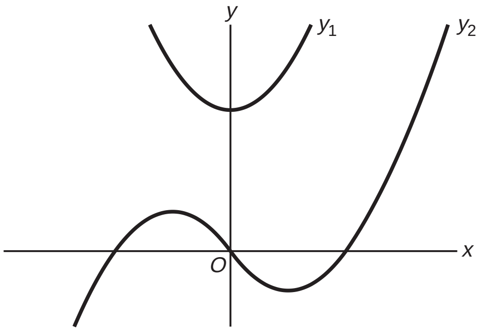
Voer in $y_1 = x^3$ en $y_2 = 3x + 1$.
De optie intersect geeft
$x \approx -1{,}53 \quad \vee \quad x \approx -0{,}35 \quad \vee \quad x \approx 1{,}88$.




Aflezen geeft $-1{,}53 < x < -0{,}35 \quad \vee \quad x > 1{,}88$.

d $x^4 + 1 < x^3 - x$

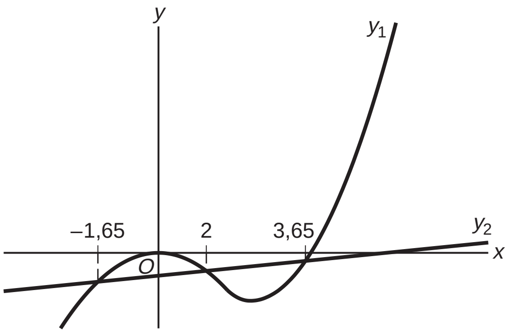
Voer in $y_1 = x^4 + 1$ en $y_2 = x^3 - x$.
De optie intersect geeft geen oplossingen.




Aflezen geeft geen oplossingen.

**34** a $x^3 - 4x^2 < 2x - 12$

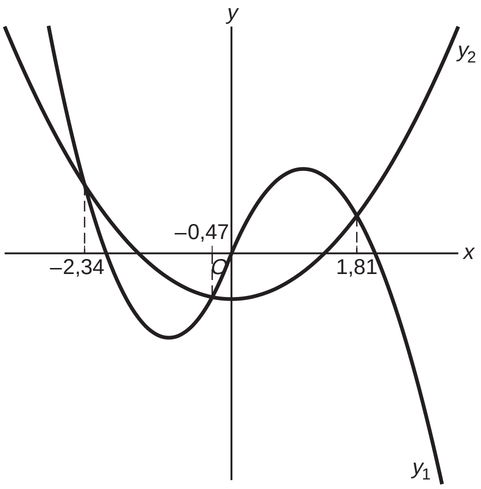
Voer in $y_1 = x^3 - 4x^2$ en $y_2 = 2x - 12$.
De optie intersect geeft
$x \approx -1{,}65 \quad \vee \quad x = 2 \quad \vee \quad x \approx 3{,}65$.




Aflezen geeft $x < -1{,}65 \quad \vee \quad 2 < x < 3{,}65$.

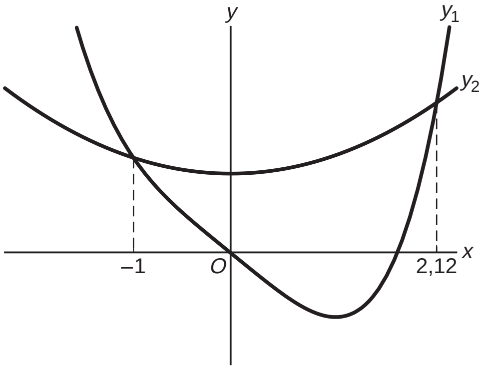
b $4x - x^3 \geq x^2 - 2$

Voer in $y_1 = 4x - x^3$ en $y_2 = x^2 - 2$.
De optie intersect geeft
$x \approx -2{,}34 \quad \vee \quad x \approx -0{,}47 \quad \vee \quad x \approx 1{,}81$.



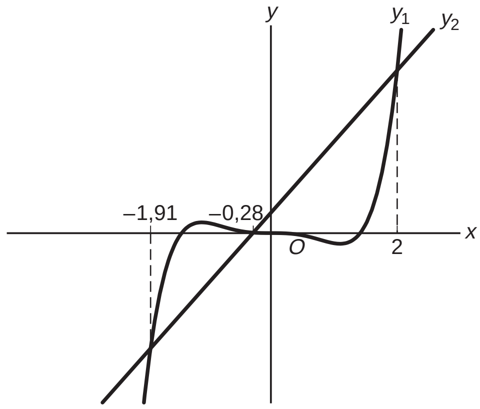

Aflezen geeft $x \leq -2{,}34 \quad \vee \quad -0{,}47 \leq x \leq 1{,}81$.

c $x^4 - 5x < x^2 + 5$
Voer in $y_1 = x^4 - 5x$ en $y_2 = x^2 + 5$.
De optie intersect geeft
$x = -1 \quad \vee \quad x \approx 2{,}12$.



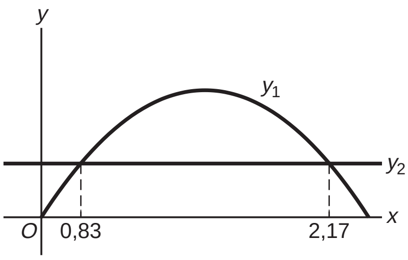

Aflezen geeft $-1 < x < 2{,}12$.

d $x^5 - 2x^3 \geq 7x + 2$
Voer in $y_1 = x^5 - 2x^3$ en $y_2 = 7x + 2$.
De optie intersect geeft
$x \approx -1{,}91 \quad \vee \quad x \approx -0{,}28 \quad \vee \quad x = 2$.



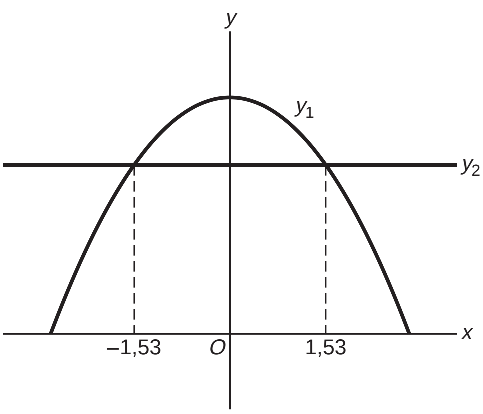

Aflezen geeft $-1{,}91 \leq x \leq -0{,}28 \quad \vee \quad x \geq 2$.

**35** $h > 9$ geeft $-5t^2 + 15t > 9$
Voer in $y_1 = -5x^2 + 15x$ en $y_2 = 9$.
De optie intersect geeft $x \approx 0{,}83 \quad \vee \quad x \approx 2{,}17$.




De bal is $2{,}17 - 0{,}83 \approx 1{,}3$ seconden hoger dan 9 m.

**36** $h < 3{,}7$ geeft $-0{,}12x^2 + 3{,}98 < 3{,}7$
Voer in $y_1 = -0{,}12x^2 + 3{,}98$ en $y_2 = 3{,}7$.
De optie intersect geeft $x \approx -1{,}53 \quad \vee \quad x \approx 1{,}53$.




De breedte is maximaal $1{,}53 - -1{,}53 \approx 3{,}05$ m.

# 5.4 Vergelijkingen gebruiken

bladzijde 169

**37** a (3, 8) invullen bij $k$ geeft $8 = 5 \cdot 3 - 7$.
Dit klopt dus ligt het punt (8, 3) op $k$.

b (2, 6) invullen bij $l$ geeft $6 = 2 \cdot a + b$
$$2a + b = 6$$

c (−1, 4) invullen bij $l$ geeft $4 = a \cdot -1 + b$
$$-a + b = 4$$

**38** $y = ax^2 + c$
(1, 8) invullen geeft $8 = a + c$ ofwel $a + c = 8$.
(2, 17) invullen geeft $17 = 4a + c$ ofwel $4a + c = 17$.

$$\begin{cases} a + c = 8 \\ 4a + c = 17 \end{cases}$$
$$\underline{\phantom{-3a = -9}} \; -$$
$$-3a = -9$$
$$\left.\begin{array}{r} a = 3 \\ a + c = 8 \end{array}\right\} \begin{array}{l} 3 + c = 9 \\ c = 5 \end{array}$$

Dus $y = 3x^2 + 5$.

bladzijde 170

**39** a $x = at^2 + b$
$t = 2$ en $x = 14$ invullen geeft $14 = 4a + b$ ofwel $4a + b = 14$.
$t = 4$ en $x = 2$ invullen geeft $2 = 16a + b$ ofwel $16a + b = 2$.

$$\begin{cases} 4a + b = 14 \\ 16a + b = 2 \end{cases}$$
$$\underline{\phantom{-12a = 12}} \; -$$
$$-12a = 12$$
$$\left.\begin{array}{r} a = -1 \\ 4a + b = 14 \end{array}\right\} \begin{array}{l} -4 + b = 14 \\ b = 18 \end{array}$$

Dus $a = -1$ en $b = 18$.

b $x = -t^2 + 18$
$t = 0$ invullen geeft $x = 18$, dus op 18 cm van de onderrand.

c $x = 0$
$-t^2 + 18 = 0$
$t^2 = 18$
$t = \sqrt{18} \approx 4{,}2$
Dus na ongeveer 4,2 seconden.

**40** $k$: $y = ax + b$ en $l$: $y = bx + a$
(2, 8) invullen bij $k$ geeft $8 = 2a + b$ ofwel $2a + b = 8$.
(2, 8) invullen bij $l$ geeft $8 = 2b + a$ ofwel $a + 2b = 8$.

$$\begin{cases} 2a + b = 8 \\ a + 2b = 8 \end{cases} \left| \begin{array}{c} 2 \\ 1 \end{array} \right| \text{ geeft } \begin{cases} 4a + 2b = 16 \\ a + 2b = 8 \end{cases}$$
$$\underline{\phantom{3a = 8}} \; -$$
$$3a = 8$$
$$\left.\begin{array}{r} a = 2\tfrac{2}{3} \\ 2a + b = 8 \end{array}\right\} \begin{array}{l} 5\tfrac{1}{3} + b = 8 \\ b = 2\tfrac{2}{3} \end{array}$$

Dus $a = 2\frac{2}{3}$ en $b = 2\frac{2}{3}$.

**41** a $y = x^2 + px + q$ snijdt $y = 2px - q$ in $(2, -1)$

$(2, -1)$ invullen in $y = x^2 + px + q$ geeft $-1 = 4 + 2p + q$ ofwel $2p + q = -5$.

$(2, -1)$ invullen in $y = 2px - q$ geeft $-1 = 4p - q$ ofwel $4p - q = -1$.

$$\begin{cases} 2p + q = -5 \\ 4p - q = -1 \end{cases}$$
$$+$$
$$6p = -6$$
$$\left.\begin{aligned} p &= -1 \\ 2p + q &= -5 \end{aligned}\right\} \begin{aligned} -2 + q &= -5 \\ q &= -3 \end{aligned}$$

Dus $p = -1$ en $q = -3$.

b $y = x^2 - x - 3$ snijden met $y = -2x + 3$.

Los op $x^2 - x - 3 = -2x + 3$

$x^2 + x - 6 = 0$

$(x + 3)(x - 2) = 0$

$x = -3 \quad \vee \quad x = 2$

$(-3, 9) \qquad (2, -1)$

Het andere snijpunt is $(-3, 9)$.

**42** $y = ax^2 + bx + c$ gaat door $(-2, -10)$, $(0, 4)$ en $(3, -5)$.

$(0, 4)$ invullen geeft $4 = c$ ofwel $c = 4$.

$(-2, -10)$ invullen geeft $-10 = 4a - 2b + 4$ ofwel $4a - 2b = -14$.

$(3, -5)$ invullen geeft $-5 = 9a + 3b + 4$ ofwel $9a + 3b = -9$.

$$\begin{cases} 4a - 2b = -14 \\ 9a + 3b = -9 \end{cases} \begin{vmatrix} 3 \\ 2 \end{vmatrix} \text{ geeft } \begin{cases} 12a - 6b = -42 \\ 18a + 6b = -18 \end{cases}$$
$$+$$
$$30a = -60$$
$$\left.\begin{aligned} a &= -2 \\ 9a + 3b &= -9 \end{aligned}\right\} \begin{aligned} -18 + 3b &= -9 \\ 3b &= 9 \\ b &= 3 \end{aligned}$$

Dus $a = -2$, $b = 3$ en $c = 4$.

**43** a ∗

b $DP = AD - AP = 20 - 5 = 15$ cm

Pythagoras: $AP^2 + AD^2 = DP^2$

$25 + AD^2 = 225$

$AD^2 = 200$

$AD = \sqrt{200} \approx 14{,}1$ cm

c $AP = 8$ geeft $DP = 20 - 8 = 12$

$AP^2 + AD^2 = DP^2$

$64 + AD^2 = 144$

$AD^2 = 80$

$AD = \sqrt{80} \approx 8{,}9$ cm

d $AP = x$ geeft $DP = AD - AP = 20 - x$

Omdat $AP = AD$ is $AD = x$.

e Pythagoras: $AP^2 + AD^2 = DP^2$

$x^2 + x^2 = (20 - x)^2$

f Voer in $y_1 = 2x^2$ en $y_2 = (20 - x)^2$.

De optie intersect geeft $x \approx 8{,}3$.

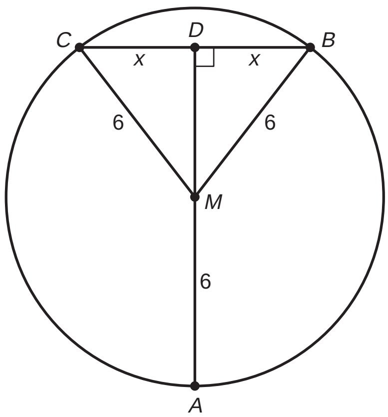
g Je moet $AP = 8{,}3$ cm nemen.

**44** a $PQ = AB - x - x = AB - 2x = 8 - 2x$
b $PS = AD - x - x = 6 - 2x$
c $\text{opp}(PQRS) = PQ \cdot PS = (8 - 2x)(6 - 2x)$
d $\text{opp}(PQRS) = 30$
$(8 - 2x)(6 - 2x) = 30$
Voer in $y_1 = (8 - 2x)(6 - 2x)$ en $y_2 = 30$.
De optie intersect geeft $x \approx 0{,}72$.
Dus $x \approx 0{,}72$ m $= 72$ cm.

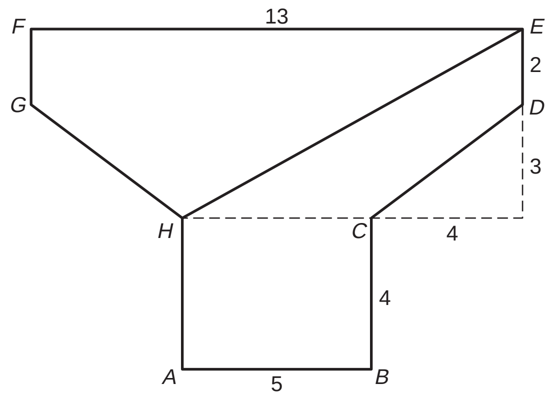
**45** Stel $AP = x$.
Nu is ook $DS = x$ en dus $AS = 7 - x$.
De oppervlakte van vierkant $PQRS$ is 25, de zijde $PS$ is dan 5.
Pythagoras: $AP^2 + AS^2 = PS^2$
$x^2 + (7 - x)^2 = 25$
$x^2 + 49 - 14x + x^2 = 25$
$2x^2 - 14x + 24 = 0$
$x^2 - 7x + 12 = 0$
$(x - 3)(x - 4) = 0$
$x = 3 \quad \vee \quad x = 4$
Dus $AP = 3 \quad \vee \quad AP = 4$.



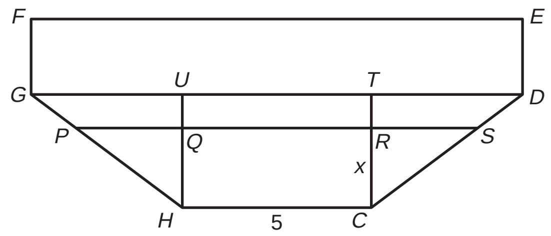
**46** Zie de figuur.
Stel $CD = BD = x$.
$AM = BM = CM = 6$
Omdat $AD = 2 \cdot BC$ krijg je $6 + DM = 4x$
$DM = 4x - 6$
Pythagoras: $DM^2 + BD^2 = BM^2$
$(4x - 6)^2 + x^2 = 36$
Voer in $y_1 = (4x - 6)^2 + x^2$ en $y_2 = 36$.
De optie intersect geeft $x \approx 2{,}82$.
De lengte van $BC = 2x \approx 5{,}65$.



**47** opp$(ABCDEH) = 5 \cdot 4 + \frac{1}{2} \cdot 9 \cdot 5 - \frac{1}{2} \cdot 4 \cdot 3$
$= 20 + 22\frac{1}{2} - 6 = 36\frac{1}{2}$




Noem de waterspiegel na het rechtopzetten van de bak $PS$.
Zie de figuur hiernaast.
Nu geldt opp$(HCSP) = 36\frac{1}{2} -$ opp$(ABCH)$
$= 36\frac{1}{2} - 5 \cdot 4 = 16\frac{1}{2}$




Stel $CR = x$
$\triangle CSR \backsim \triangle CDT$
$\frac{CR}{CT} = \frac{RS}{DT}$
$\frac{x}{3} = \frac{RS}{4}$
$3RS = 4x$
$RS = \frac{4}{3}x$
Dus $PS = PQ + QR + RS = \frac{4}{3}x + 5x + \frac{4}{3}x = \frac{8}{3}x + 5.$
opp$(HCSP) = \frac{1}{2}(HC + PS) \cdot CR = \frac{1}{2}(5 + \frac{8}{3}x + 5) \cdot x = \frac{1}{2}x \cdot (\frac{8}{3}x + 10) = \frac{4}{3}x^2 + 5x$
Voer in $y_1 = \frac{4}{3}x^2 + 5x$ en $y_2 = 16\frac{1}{2}$.
De optie intersect geeft $x \approx 2{,}11$.
Dus het water staat $4 + x \approx 4 + 2{,}11$ dm $\approx 6{,}1$ dm hoog.

bladzijde 173

**48** a De vermenigvuldigingsfactor $k = 3$.
Dus $I\,(T\,ABCD) = 27 \cdot I\,(T\,PQRS)$.
Van de piramide zit $\frac{1}{27}$ deel buiten en $\frac{26}{27}$ deel in de kubus.

b $I\,(T\,ABCD) \xrightarrow{\times k^3} I\,(T\,PQRS)$
dus $k^3 = \frac{1}{3}$
$k = \sqrt[3]{\frac{1}{3}} \approx 0{,}69$
Stel de hoogte van $T\,PQRS$ is $x$, dan is de hoogte van $T\,ABCD$ gelijk aan $x + 6$.
De hoogte van $T\,PQRS$ is ongeveer 0,69 keer de hoogte van $T\,ABCD$.
dus $x \approx 0{,}69(x + 6)$
$x \approx 0{,}69x + 4{,}16$
$0{,}31x \approx 4{,}16$
$x \approx 13{,}6$
De hoogte van $T\,PQRS$ is 13,6 cm.

c $I(T\,ABCD) \xrightarrow{\times k^3} I(T\,PQRS)$

dus $k^3 = \frac{1}{4}$

$k = \sqrt[3]{\frac{1}{4}} \approx 0{,}63$

Stel de hoogte van $T\,PQRS$ is $x$, dan is de hoogte van $T\,ABCD$ gelijk aan $x + 6$.

De hoogte van $T\,PQRS$ is ongeveer 0,63 keer de hoogte van $T\,ABCD$.

dus $x \approx 0{,}63(x + 6)$

$x \approx 0{,}63x + 3{,}78$

$0{,}37x \approx 3{,}78$

$x \approx 10{,}21$

De hoogte van $T\,PQRS$ is 10,2 cm.

bladzijde 174

**49** $I$ (piramide) $\xrightarrow{\times k^3}$ $I$ (deel van de piramide binnen de kubus)

dus $k^3 = \frac{1}{4}$

$k = \sqrt[3]{\frac{1}{4}} \approx 0{,}63$

Stel de hoogte van het deel buiten de kubus $x$, dan is de hoogte van de hele piramide gelijk aan $x + 10$.

De hoogte van het deel binnen de kubus is ongeveer 0,63 keer de hoogte van de hele piramide.

Dus $10 \approx 0{,}63(x + 10)$

$10 \approx 0{,}63x + 6{,}3$

$3{,}7 \approx 0{,}63x$

$x \approx 5{,}9$

De hoogte van de piramide is ongeveer $10 + 5{,}9 = 15{,}9$.

**50** $I$ (kegel) $\xrightarrow{\times k^3}$ $I$ (deel van de kegel buiten de kubus)

dus $k^3 = \frac{1}{5}$

$k = \sqrt[3]{\frac{1}{5}} \approx 0{,}58$

Stel de hoogte van het deel buiten de kubus $x$, dan is de hoogte van de hele kegel gelijk aan $x + 6$.

De hoogte van het deel buiten de kubus is ongeveer 0,58 keer de hoogte van de hele kegel.

Dus $x \approx 0{,}58(x + 6)$

$x \approx 0{,}58x + 3{,}51$

$0{,}42x \approx 3{,}51$

$x \approx 8{,}45$

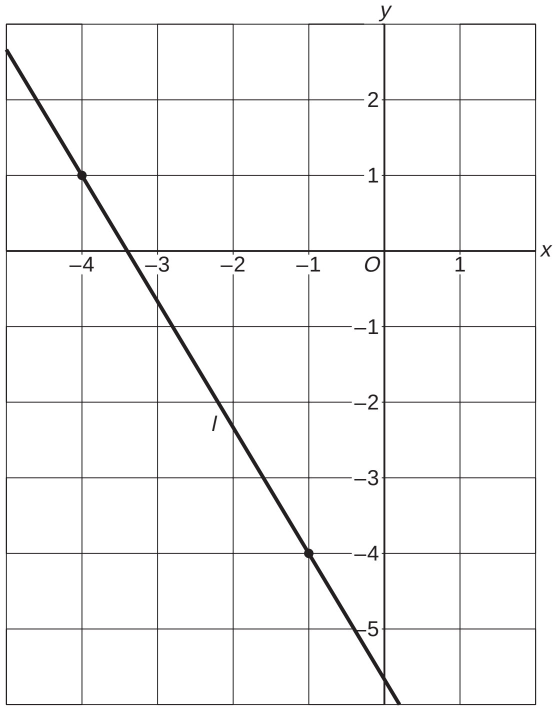
De hoogte van de kegel is ongeveer $x + 6 \approx 14{,}45$.

# Diagnostische toets

bladzijde 178

**1** a $l: 5x + 3y = -17$

| $x$ | $-4$ | $-1$ |
|---|---|---|
| $y$ | $1$ | $-4$ |




b $A\,(-25, 36)$ invullen bij $l$ geeft $5 \cdot -25 + 3 \cdot 36 = -17$
$-125 + 108 = -17$
Klopt, dus $A$ ligt op $l$.

$B\,(60, -105)$ invullen bij $l$ geeft $5 \cdot 60 + 3 \cdot -105 = -17$
$300 - 315 = -17$
Klopt niet, dus $B$ ligt niet op $l$.

$C\,(83, -144)$ invullen bij $l$ geeft $5 \cdot 83 + 3 \cdot -144 = -17$
$415 - 432 = -17$
Klopt, dus $C$ ligt op $l$.

c $(-56, q)$ invullen bij $l$ geeft $5 \cdot -56 + 3 \cdot q = -17$
$-280 + 3q = -17$
$3q = 263$
$q = 87\frac{2}{3}$

**2** a
$$\begin{cases} 2x - 5y = -9 \\ 3x + 5y = 24 \end{cases} \quad +$$
$$5x = 15$$
$$\left.\begin{array}{l} x = 3 \\ 3x + 5y = 24 \end{array}\right\} \quad \begin{array}{l} 9 + 3y = 24 \\ 5y = 15 \\ y = 3 \end{array}$$
De oplossing is $(3, 3)$.

b
$$\begin{cases} 4x + 2y = 4 \\ 4x - 7y = 40 \end{cases} \quad -$$
$$9y = -36$$
$$\left.\begin{array}{l} y = -4 \\ 4x + 2y = 4 \end{array}\right\} \quad \begin{array}{l} 4x - 8 = 4 \\ 4x = 12 \\ x = 3 \end{array}$$
De oplossing is $(3, -4)$.

c
$$\begin{cases} 4x + 5y = 27 \\ -2x + 3y = 25 \end{cases} \begin{vmatrix} 1 \\ 2 \end{vmatrix} \text{ geeft } \begin{cases} 4x + 5y = 27 \\ -4x + 6y = 50 \end{cases} \quad +$$
$$11y = 77$$
$$\left.\begin{array}{l} y = 7 \\ 4x + 5y = 27 \end{array}\right\} \quad \begin{array}{l} 4x + 35 = 27 \\ 4x = -8 \\ x = -2 \end{array}$$
De oplossing is $(-2, 7)$.

d
$$\begin{cases} 6x - 3y = 19 \\ -4x - 6y = 14 \end{cases} \begin{vmatrix} 2 \\ 1 \end{vmatrix} \text{ geeft } \begin{cases} 12x - 6y = 38 \\ -4x - 6y = 14 \end{cases} \quad -$$
$$16x = 24$$
$$\left.\begin{array}{l} x = 1\frac{1}{2} \\ 6x - 3y = 19 \end{array}\right\} \quad \begin{array}{l} 9 - 3y = 19 \\ -3y = 10 \\ y = -3\frac{1}{3} \end{array}$$
De oplossing is $(1\frac{1}{2}, -3\frac{1}{3})$.

**3**
$$\begin{cases} 5x - 6y = -27 \\ 15x + 8y = 10 \end{cases} \begin{vmatrix} 3 \\ 1 \end{vmatrix} \text{ geeft } \begin{cases} 15x - 18y = -81 \\ 15x + 8y = 10 \end{cases} \quad -$$
$$-26y = -91$$
$$\left.\begin{array}{l} y = 3\frac{1}{2} \\ 5x - 6y = -27 \end{array}\right\} \quad \begin{array}{l} 5x - 21 = -27 \\ 5x = -6 \\ x = -1\frac{1}{5} \end{array}$$
Het snijpunt is $(-1\frac{1}{5}, 3\frac{1}{2})$.

**4** a $\begin{cases} 5x - 3y = 3 \\ y = \frac{2}{3}x - 4 \end{cases}$ substitueren geeft $5x - 3(\frac{2}{3}x - 4) = 3$

$5x - 2x + 12 = 3$

$3x = -9$

$\left.\begin{array}{l} x = -3 \\ y = \frac{2}{3}x - 4 \end{array}\right\}$ $y = \frac{2}{3} \cdot -3 - 4 = -6$

De oplossing is $(-3, -6)$.

b $\begin{cases} x = 1{,}4y - 3 \\ -5x + 6y = 8 \end{cases}$ substitueren geeft $-5(1{,}4y - 3) + 6y = 8$

$-7y + 15 + 6y = 8$

$-y = -7$

$\left.\begin{array}{l} y = 7 \\ x = 1{,}4y - 3 \end{array}\right\}$ $x = 1{,}4 \cdot 7 - 3 = 6{,}8$

De oplossing is $(6{,}8; 7)$.

**5** a $x^8 = 256$

$x = \sqrt[8]{256} = 2 \quad \vee \quad x = -\sqrt[8]{256} = -2$

b $x^3 = -216$

$x = \sqrt[3]{-216} = -6$

c $4x^4 + 8 = 7$

$4x^4 = -1$

$x^4 = -\frac{1}{4}$

geen oplossing

d $5 - x^5 = -4$

$-x^5 = -9$

$x^5 = 9$

$x = \sqrt[5]{9}$

e $9(x - 1)^4 = 144$

$(x - 1)^4 = 16$

$x - 1 = 2 \quad \vee \quad x - 1 = -2$

$x = 3 \quad \vee \quad x = -1$

f $\frac{1}{4}(2x - 1)^7 - 12 = -44$

$\frac{1}{4}(2x - 1)^7 = -32$

$(2x - 1)^7 = -128$

$2x - 1 = -2$

$2x = -1$

$x = -\frac{1}{2}$

**6** a $5x^3 - 1 = 9$

$5x^3 = 10$

$x^3 = 2$

$x = \sqrt[3]{2} \approx 1{,}260$

b $\frac{1}{2}(x - 1)^4 = 12$

$(x - 1)^4 = 24$

$x - 1 = \sqrt[4]{24} \quad \vee \quad x - 1 = -\sqrt[4]{24}$

$x = 1 + \sqrt[4]{24} \approx 3{,}213 \quad \vee \quad x = 1 - \sqrt[4]{24} \approx -1{,}213$

c $(1 - 2x)^5 - 4 = 12$

$(1 - 2x)^5 = 16$

$1 - 2x = \sqrt[5]{16}$

$-2x = -1 + \sqrt[5]{16}$

$x = \frac{1}{2} - \frac{1}{2}\sqrt[5]{16} \approx -0{,}371$

**7** a $I$ (grote kogel) $= 10\,000 I$ (kleine kogel)

dus $k^3 = 10\,000$

$k = \sqrt[3]{10\,000}$

diameter grote kogel $= \sqrt[3]{10\,000} \cdot 3 \approx 65$ mm

b $k^2 = (\sqrt[3]{10\,000})^2 \approx 464$

Dus de oppervlakte is ongeveer 464 keer zo groot.

c $k^2 = 1000$

$k = \sqrt{1000} \approx 31{,}6$

$k^3 \approx 31\,620$

Dus $n = 31\,620$.

**8** a $5(7-x) > 3(2x-3)$

$35 - 5x > 6x - 9$

$-11x > -44$

$x < 4$

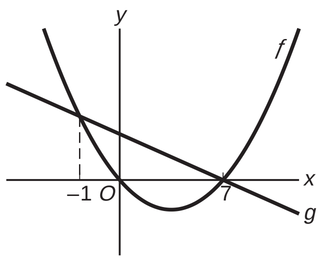
b $\underbrace{x^2 - 7x}_{f(x)} < \underbrace{-x + 7}_{g(x)}$

$x^2 - 7x = -x + 7$

$x^2 - 6x - 7 = 0$

$(x-7)(x+1) = 0$

$x = 7 \vee x = -1$




Aflezen geeft $-1 < x < 7$.

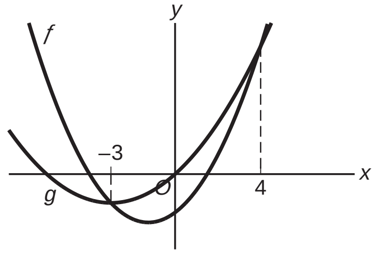
c $\underbrace{2x^2 + 5x - 12}_{f(x)} \geq \underbrace{x^2 + 6x}_{g(x)}$

$2x^2 + 5x - 12 = x^2 + 6x$

$x^2 - x - 12 = 0$

$(x-4)(x+3) = 0$

$x = 4 \vee x = -3$




Aflezen geeft $x \leq -3 \vee x \geq 4$.

d $\underbrace{x^2 - 9}_{f(x)} \geq \underbrace{2x^2 + 11x + 9}_{g(x)}$

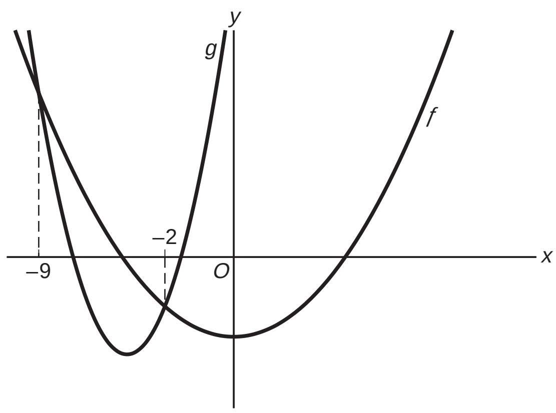
$x^2 - 9 = 2x^2 + 11x + 9$

$-x^2 - 11x - 18 = 0$

$x^2 + 11x + 18 = 0$

$(x+2)(x+9) = 0$

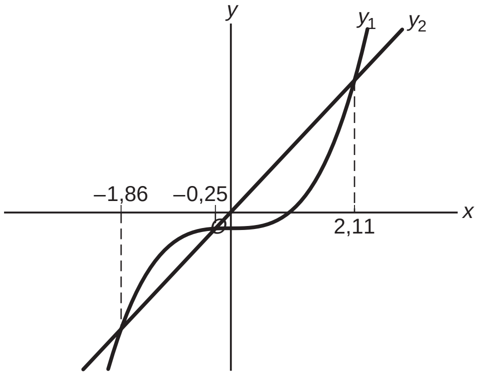
$x = -2 \vee x = -9$




Aflezen geeft $-9 \leq x \leq -2$.

**9** a $x^3 - 1 > 4x$

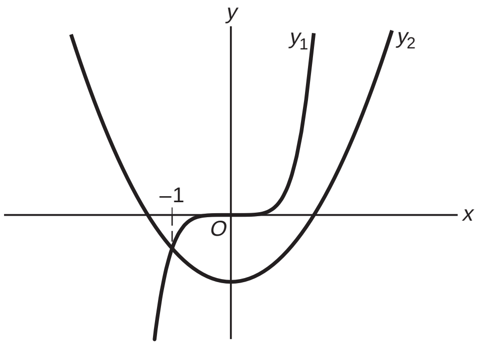
Voer in $y_1 = x^3 - 1$ en $y_2 = 4x$.
De optie intersect geeft
$x \approx -1{,}86 \;\vee\; x \approx -0{,}25 \;\vee\; x \approx 2{,}11$.




Aflezen geeft
$-1{,}86 < x < -0{,}25 \;\vee\; x > 2{,}11$.

b $x^5 < x^2 - 2$

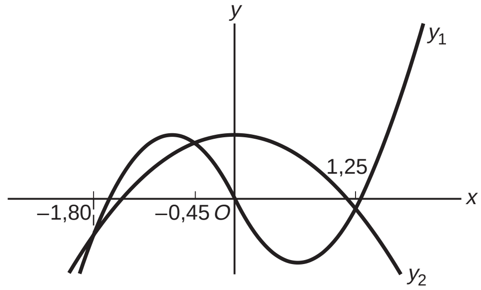
Voer in $y_1 = x^5$ en $y_2 = x^2 - 2$.
De optie intersect geeft $x = -1$.




Aflezen geeft $x < -1$.

c $x^3 - 2x < -x^2 + 1$

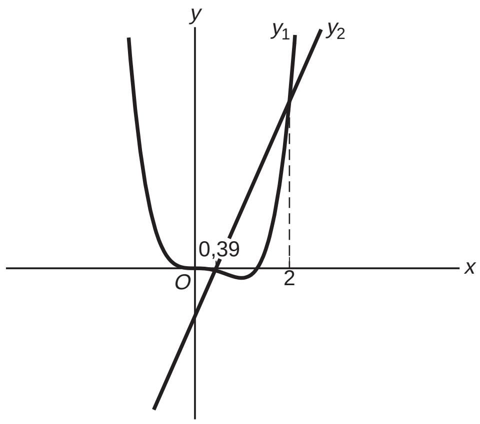
Voer in $y_1 = x^3 - 2x$ en $y_2 = -x^2 + 1$.
De optie intersect geeft
$x \approx -1{,}80 \;\vee\; x \approx -0{,}45 \;\vee\; x \approx 1{,}25$.




Aflezen geeft
$x < -1{,}80 \;\vee\; -0{,}45 < x < 1{,}25$.

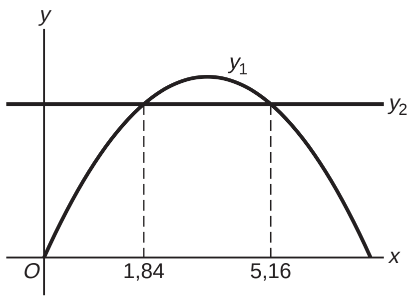
d $x^4 - x^3 > 5x - 2$

Voer in $y_1 = x^4 - x^3$ en $y_2 = 5x - 2$.
De optie intersect geeft $x \approx 0{,}39 \;\vee\; x = 2$.




Aflezen geeft $x < 0{,}39 \;\vee\; x > 2$.

**10** $h > 40$ geeft $-4t^2 + 28t + 2 > 40$
Voer in $y_1 = -4x^2 + 28x + 2$ en $y_2 = 40$.
De optie intersect geeft $x \approx 1{,}84 \;\vee\; x \approx 5{,}16$.
De pijl is $5{,}16 - 1{,}84 \approx 3{,}3$ seconden hoger dan 40 m.




**11** $y = ax^2 - 5x + c$
$(-3, -6)$ invullen geeft $-6 = 9a + 15 + c$
ofwel $9a + c = -21$.
$(1, -2)$ invullen geeft $-2 = a - 5 + c$ ofwel $a + c = 3$.

$$\begin{cases} 9a + c = -21 \\ a + c = 3 \end{cases} \quad -$$

$$8a = -18$$

$$\left.\begin{array}{l} a = -2\tfrac{1}{4} \\ a + c = 3 \end{array}\right\} \quad \begin{array}{l} -2\tfrac{1}{4} + c = 3 \\ c = 5\tfrac{1}{4} \end{array}$$

Dus $a = -2\frac{1}{4}$ en $c = 5\frac{1}{4}$.

**12** $y = ax + b$

$(-1, 4)$ invullen geeft $4 = -a + b$ ofwel $a - b = -4$.

$(5, -8)$ invullen geeft $-8 = 5a + b$ ofwel $5a + b = -8$.

$$\begin{cases} a - b = -4 \\ 5a + b = -8 \end{cases} \quad +$$

$$6a = -12$$

$$\left.\begin{array}{l} a = -2 \\ a - b = -4 \end{array}\right\} \quad \begin{array}{l} -2 - b = -4 \\ -b = -2 \\ b = 2 \end{array}$$

$y = px^2 + q$

$(-1, 4)$ invullen geeft $4 = p + q$ ofwel $p + q = 4$.

$(5, -8)$ invullen geeft $-8 = 25p + q$ ofwel $25p + q = -8$.

$$\begin{cases} p + q = 4 \\ 25p + q = -8 \end{cases} \quad -$$

$$-24p = 12$$

$$\left.\begin{array}{l} p = -\frac{1}{2} \\ p + q = 4 \end{array}\right\} \quad \begin{array}{l} -\frac{1}{2} + q = 4 \\ q = 4\frac{1}{2} \end{array}$$

Dus $a = -2$, $b = 2$, $p = -\frac{1}{2}$ en $q = 4\frac{1}{2}$.

**13** Stel de breedte $x$. De lengte is dan $2x$.

Pythagoras: $(2x)^2 + x^2 = 8^2$

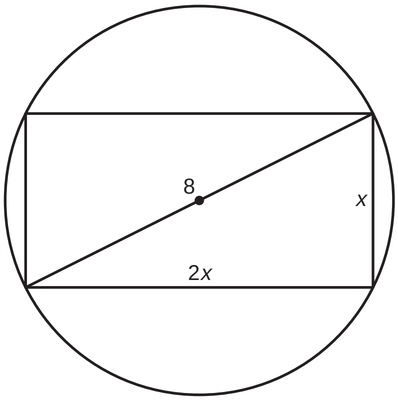
$4x^2 + x^2 = 64$

$5x^2 = 64$

$x^2 = \frac{64}{5}$

$x \approx 3{,}58$ cm

De rechthoek is dus 7,2 cm lang en 3,6 cm breed.




**14** $I$ (bovenste deel) $= 0{,}35I$ (hele kegel)

dus $k^3 = 0{,}35$

$k = \sqrt[3]{0{,}35} \approx 0{,}705$

Stel de hoogte van het bovenste deel $x$.

Dus $0{,}705(x + 10) \approx x$

$0{,}705x + 7{,}05 \approx x$

$7{,}05 \approx 0{,}295x$

$x \approx 23{,}9$ cm

De kegel is ongeveer $23{,}9 + 10 = 33{,}9$ cm hoog.