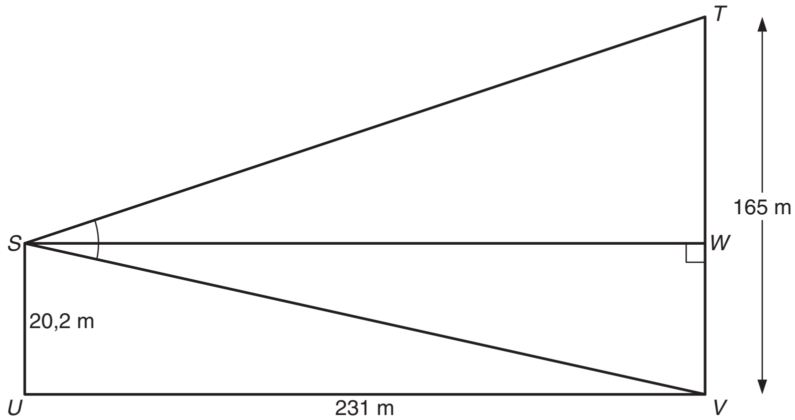
Diagnostische toets

bladzijde 149

1



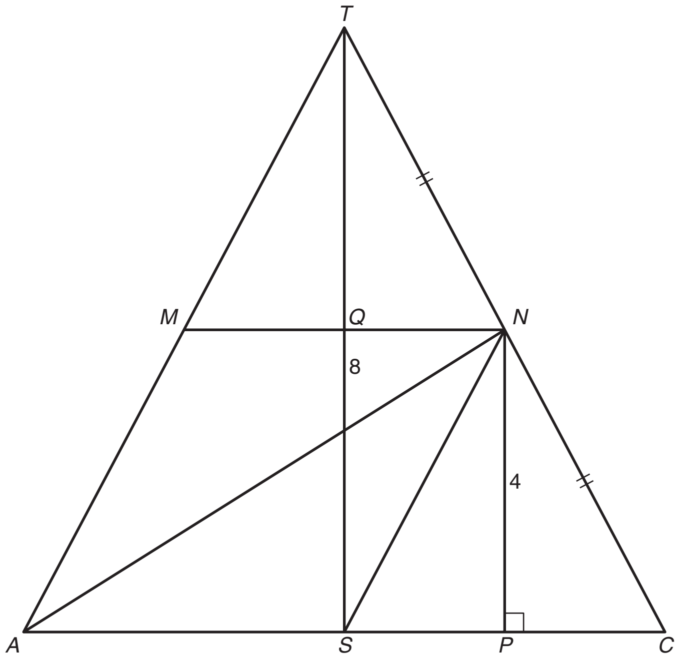
De gevraagde hoek is $\angle VST$.

In $\triangle VSW$: $\tan \angle VSW = \dfrac{VW}{SW} = \dfrac{20{,}2}{231}$ geeft $\angle VSW \approx 5{,}0°$

In $\triangle WST$: $\tan \angle WST = \dfrac{WT}{SW} = \dfrac{165 - 20{,}2}{231}$ geeft $\angle WST \approx 32{,}1°$

$\angle VST = \angle VSW + \angle WST \approx 5{,}0° + 32{,}1° \approx 37°$

2 a



In $\triangle ABC$ geeft de stelling van Pythagoras
$AC^2 = 6^2 + 6^2 = 72$, dus $AC = \sqrt{72}$.
Teken in hulpvlak ACT het lijnstuk NP loodrecht op AC.
N is het midden van CT, dus $NP = \frac{1}{2} \cdot TS = 4$.
$AS = SC = \frac{1}{2}AC = \frac{1}{2}\sqrt{72}$
en $SP = PC = \frac{1}{2} \cdot SC = \frac{1}{4}\sqrt{72}$
Dus $AP = AS + SP = \frac{3}{4}\sqrt{72}$.

In $\triangle APN$

$\tan \angle PAN = \dfrac{NP}{AP} = \dfrac{4}{\frac{3}{4}\sqrt{72}}$

dus $\angle NAC = \angle PAN \approx 32^\circ$.

Trek MN en noem het snijpunt met TS punt Q en trek NS.

Q is het midden van TS, dus $QS = 4$.

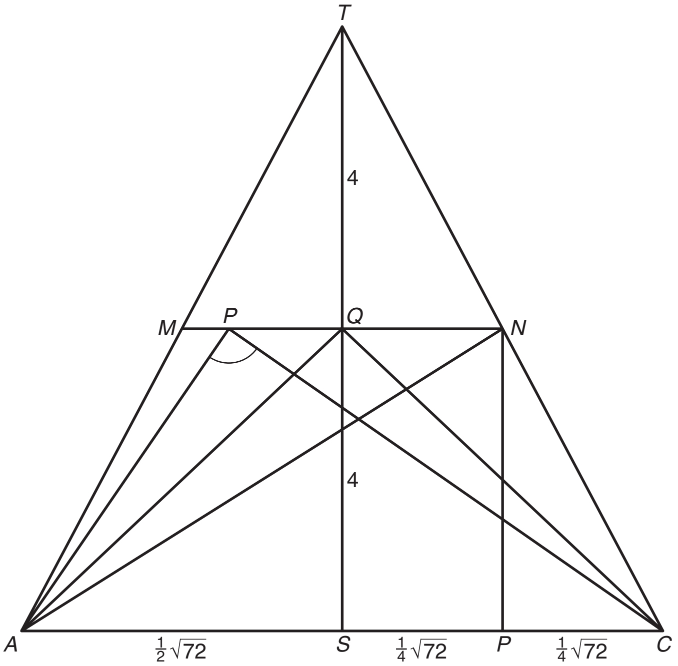
$MN = \frac{1}{2} \cdot AC = \frac{1}{2}\sqrt{72}$

$QN = \frac{1}{2}MN = \frac{1}{4}\sqrt{72}$

In $\triangle SNQ$: $\tan \angle MNS = \dfrac{QS}{QN} = \dfrac{4}{\frac{1}{4}\sqrt{72}}$

$\angle MNS \approx 62^\circ$

b



$\angle APC$ is maximaal als $P = Q$.

In $\triangle AQS$: $\tan \angle AQS = \dfrac{AS}{QS} = \dfrac{\frac{1}{2}\sqrt{72}}{4}$ geeft $\angle AQS \approx 46{,}7^\circ$.

Dus $\angle AQC = 2 \cdot \angle AQS \approx 93^\circ$.

$\angle APC$ is minimaal als $P = N$ of als $P = M$.

In $\triangle ANP$: $\tan \angle ANP = \dfrac{AP}{NP} = \dfrac{\frac{3}{4}\sqrt{72}}{4}$ geeft $\angle ANP \approx 57{,}8^\circ$

In $\triangle PNC$: $\tan PNC = \dfrac{PC}{NP} = \dfrac{\frac{1}{4}\sqrt{72}}{4}$ geeft $\angle PNC \approx 27{,}9^\circ$

Dus $\angle ANC = \angle ANP + \angle PNC \approx 86^\circ$.

Dus $\angle APC$ kan waarden aannemen tussen 86° en 93°.

3 Cosinusregel

$a^2 = b^2 + c^2 - 2bc\cos\alpha$

$3^2 = 6^2 + 8^2 - 2 \cdot 6 \cdot 8 \cdot \cos\alpha$

$9 = 36 + 64 - 96\cos\alpha$

$96\cos\alpha = 91$

$\cos\alpha = \dfrac{91}{96}$

$\alpha \approx 18{,}6^\circ \approx 19^\circ$

$b^2 = a^2 + c^2 - 2ac\cos\beta$

$6^2 = 3^2 + 8^2 - 2 \cdot 3 \cdot 8 \cdot \cos\beta$
$36 = 9 + 64 - 48 \cdot \cos\beta$
$48\cos\beta = 37$
$\cos\beta = \frac{37}{48}$
$\beta \approx 39{,}6^\circ \approx 40^\circ$
$\gamma = 180^\circ - \alpha - \beta = 180^\circ - 18{,}6^\circ - 39{,}6^\circ \approx 121{,}9^\circ \approx 122^\circ$

4 Cosinusregel
$b^2 = a^2 + c^2 - 2ac\cos\beta$
$b^2 = 6^2 + 8^2 - 2 \cdot 6 \cdot 8 \cdot \cos 55^\circ$
$b^2 = 100 - 96 \cdot \cos 55^\circ$
$b^2 \approx 44{,}94$
$b \approx \sqrt{44{,}94} \approx 6{,}7$

5 a In zijvlak $ABFE$: $BE^2 = AB^2 + AE^2$
$BE^2 = 49 + 16 = 65$
$BE = \sqrt{65}$
In zijvlak $BCGF$: $BG^2 = BC^2 + CG^2 = 36 + 16 = 52$
$BG = \sqrt{52}$
In voorvak: $EG^2 = EH^2 + HG^2 = 36 + 49 = 85$
$EG = \sqrt{85}$

b In $\triangle BEG$ de cosinusregel $EG^2 = BE^2 + BG^2 - 2 \cdot BE \cdot BG\cos\angle EBG$
$85 = 65 + 52 - 2 \cdot \sqrt{65} \cdot \sqrt{52} \cdot \cos\angle EBG$
$2 \cdot \sqrt{65} \cdot \sqrt{52} \cdot \cos\angle EBG = 32$
$\cos\angle EBG = \frac{32}{2\sqrt{65} \cdot \sqrt{52}}$
$\angle EGB \approx 74{,}0^\circ$

c $O(\triangle EBG) = \frac{1}{2} \cdot BE \cdot BG \cdot \sin\angle EBG$
$\approx \frac{1}{2} \cdot \sqrt{65} \cdot \sqrt{52} \cdot \sin 74{,}0^\circ$
$\approx 27{,}9$

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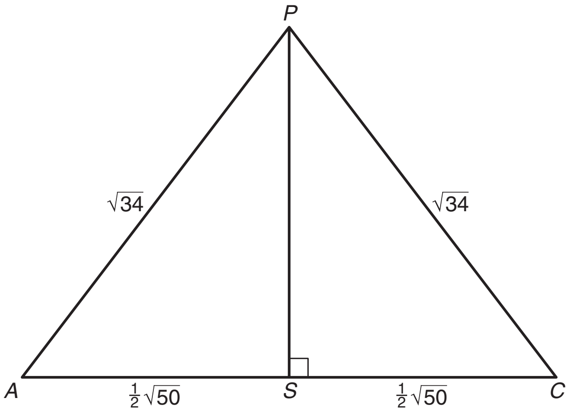
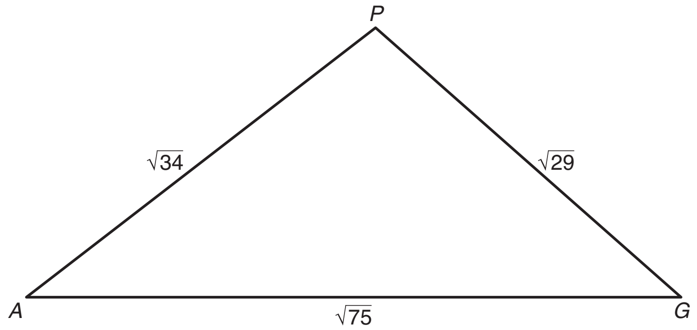
6 a $\angle(AP, CP) = \angle APC$ in $\triangle APC$.
$AP = \sqrt{5^2 + 3^2} = \sqrt{34}$
$PC = \sqrt{5^2 + 3^2} = \sqrt{34}$
$AC = \sqrt{5^2 + 5^2} = \sqrt{50}$
$\sin\angle APS = \frac{\frac{1}{2}\sqrt{50}}{\sqrt{34}}$
$\angle APS \approx 37{,}3^\circ$
$\angle(AP, CP) = \angle APC = 2 \cdot \angle APS \approx 75^\circ$

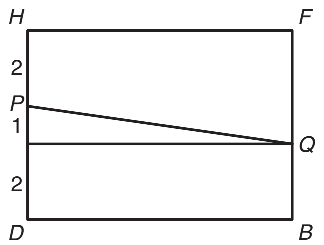


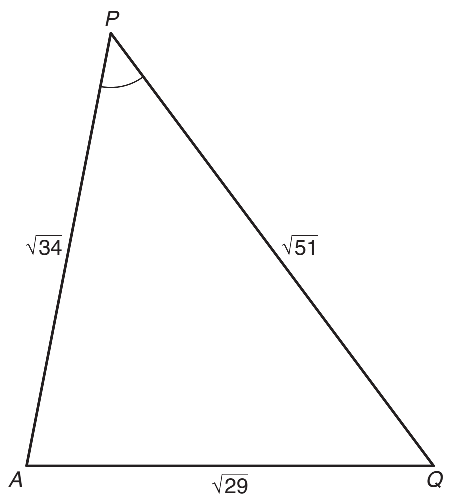
b $\angle(APG)$ in $\triangle APG$.
$PG = \sqrt{5^2 + 2^2} = \sqrt{29}$
$AG = \sqrt{AC^2 + CG^2} = \sqrt{50 + 25} = \sqrt{75}$



$AG^2 = AP^2 + GP^2 - 2 \cdot AP \cdot GP \cdot \cos \angle P$
$75 = 34 + 29 - 2 \cdot \sqrt{34} \cdot \sqrt{29} \cdot \cos \angle P$
$2 \cdot \sqrt{34} \cdot \sqrt{29} \cdot \cos \angle P = -12$
$\cos \angle P = \dfrac{-12}{2 \cdot \sqrt{34} \cdot \sqrt{29}}$
$\angle P \approx 101^\circ$
Dus $\angle(AP, PG) = 180^\circ - \angle P \approx 79^\circ$.

c $\angle(AP, PQ) = \angle APQ$ in $\triangle APQ$:
$AP = \sqrt{34}$; $AQ = \sqrt{5^2 + 2^2} = \sqrt{29}$
$DB = \sqrt{50}$; $PQ = \sqrt{50 + 1^2} = \sqrt{51}$





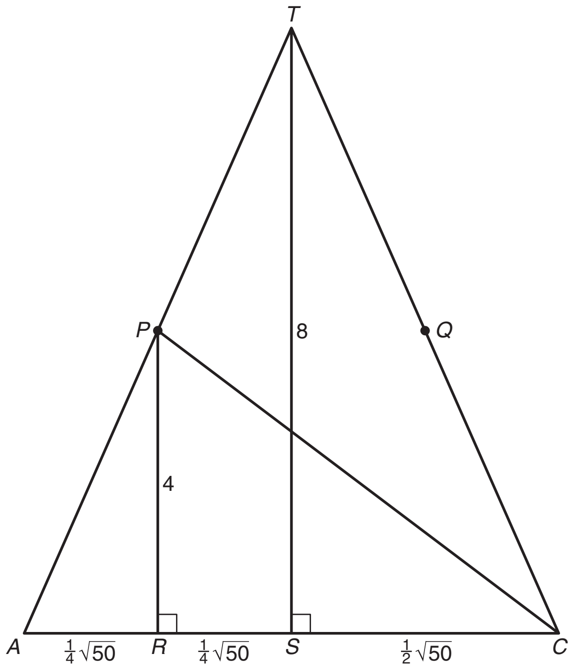
Cosinusregel
$AQ^2 = AP^2 + QP^2 - 2 \cdot AP \cdot QP \cdot \cos \angle APQ$
$29 = 34 + 51 - 2 \cdot \sqrt{34} \cdot \sqrt{51} \cdot \cos \angle APQ$
$2 \cdot \sqrt{34} \cdot \sqrt{51} \cdot \cos \angle APQ = 56$
$\cos \angle APQ = \dfrac{56}{2\sqrt{34} \cdot \sqrt{51}} \approx 0{,}672$
$\angle APQ \approx 48^\circ$
Dus $\angle(AP, PQ) \approx 48^\circ$.

d $AQ = \sqrt{29}$; $QG = \sqrt{34}$; $AG = \sqrt{75}$
Cosinusregel
$AG^2 = AQ^2 + QG^2 - 2 \cdot AQ \cdot QG \cdot \cos \angle GQA$
$75 = 29 + 34 - 2 \cdot \sqrt{29} \cdot \sqrt{34} \cos \angle GQA$
$2 \cdot \sqrt{29} \cdot \sqrt{34} \cdot \cos \angle GQA = -12$
$\cos \angle GQA = -\dfrac{12}{2\sqrt{29} \cdot \sqrt{34}} \approx -0{,}191$
$\angle GQA \approx 101^\circ$

7 a $\angle(PQ, DBT) = 90°$ want $PQ /\!/ AC$ en $AC \perp DBT$.

b $\angle(CP, DBT) = \angle(CP, TS) = \angle(CP, PR) = \angle CPR$

$AC = \sqrt{5^2 + 5^2} = \sqrt{50}$, dus $SC = \frac{1}{2}\sqrt{50}$ en $RS = \frac{1}{4}\sqrt{50}$.



$\tan \angle CPR = \dfrac{\frac{3}{4}\sqrt{50}}{4}$

$\angle CPR \approx 53{,}0°$

Dus $\angle(CP, DBT) \approx 53°$.

c De hellingshoek van AQ = de hellingshoek van $CP = \angle PCA$.

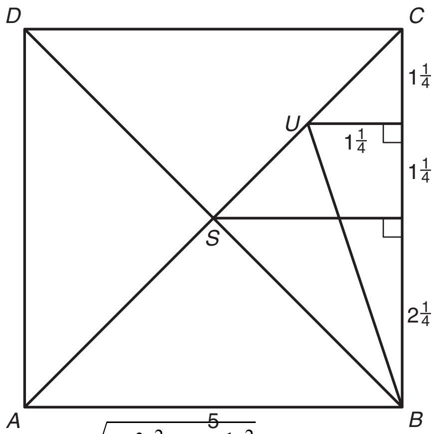
Zie de figuur bij b.

$\angle PCA = 90° - \angle CPR = 90° - 53° = 37°$

Dus de hellingshoek van AQ is 37°.

d Noem het midden van SC punt U.

De hellingshoek van $BQ = \angle(BQ, BU) = \angle QBU$.



$BU = \sqrt{(3\frac{3}{4})^2 + (1\frac{1}{4})^2} \approx 3{,}95$

$\tan \angle QBU = \dfrac{QU}{BU} = \dfrac{4}{3{,}95}$

$\angle QBU \approx 45{,}3°$

Dus de hellingshoek van BQ is 45°.

8 a $\angle(ABK, ABC)$ in standvlak $BCKJ$.

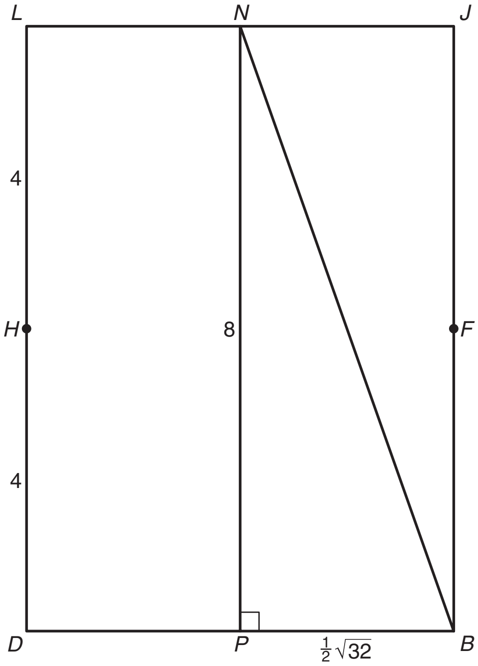
$\tan \angle KBC = \frac{8}{4} = 2$

$\angle KBC \approx 63{,}4°$

Dus $\angle(ABK, ABC) \approx 63°$.

b De hellingshoek van $BIK = \angle NBD$ waarbij N het snijpunt is van IK en LJ

$DB = \sqrt{4^2 + 4^2} = \sqrt{32}$

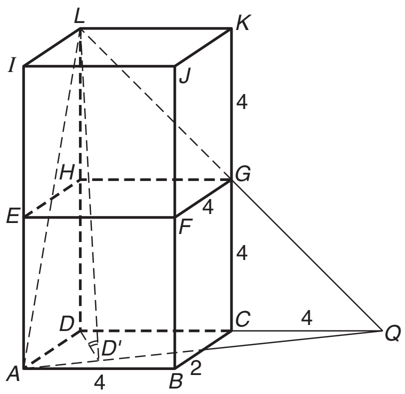


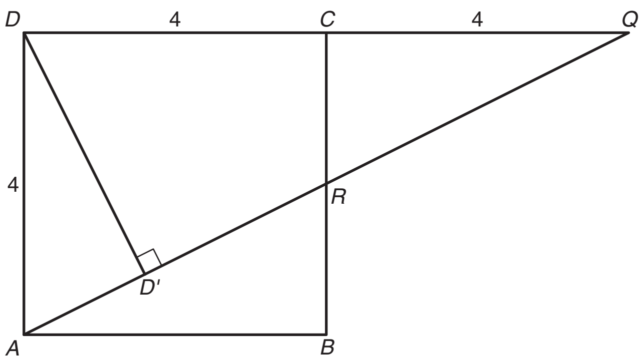
$$\tan \angle NBD = \frac{8}{\frac{1}{2}\sqrt{32}}$$

$\angle NBD \approx 70{,}53°$

De hellingshoek van BIK is dus ongeveer 71°.

c





Trek DD' loodrecht op AQ.
$AQ = \sqrt{4^2 + 8^2} = \sqrt{80}$
In $\triangle ADQ$ de zijde × hoogte-methode: $AQ \times DD' = AD \times DQ$

$$\sqrt{80} \times DD' = 4 \times 8$$

$$DD' = \frac{32}{\sqrt{80}} \approx 3{,}58$$

$$\tan \angle LD'D = \frac{8}{3{,}58}$$

$$\angle LD'D \approx 65{,}9^\circ$$

Dus de hellingshoek van AGL is ongeveer 66°.

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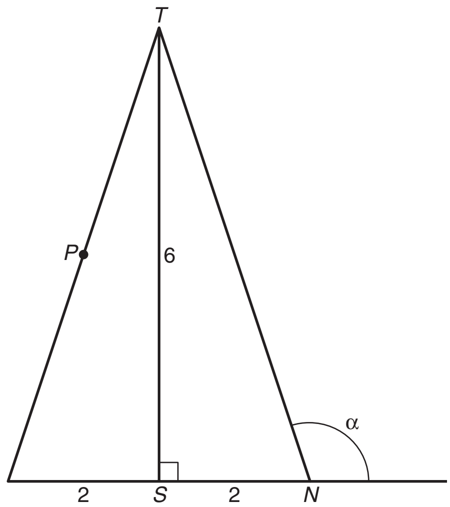
9 a Verticaal hulpvlak door T en het midden N van BC.

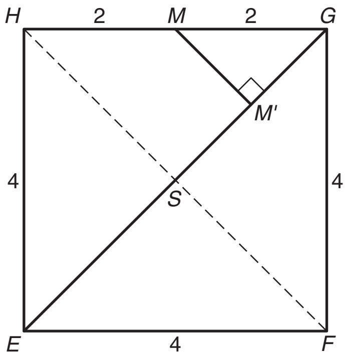


$\tan \angle TNS = \frac{6}{2} = 3$
$\angle TNS \approx 71{,}6^\circ$
$\alpha = 180^\circ - \angle TNS \approx 108{,}4^\circ \approx 108^\circ$

b Lengte $= \frac{108{,}4}{360} \cdot 2\pi \cdot NT = \frac{180{,}4}{360} \cdot 2\pi \cdot \sqrt{2^2 + 6^2} \approx 11{,}97$
De lengte van de rotatiebaan is dus ongeveer 12,0 dm.

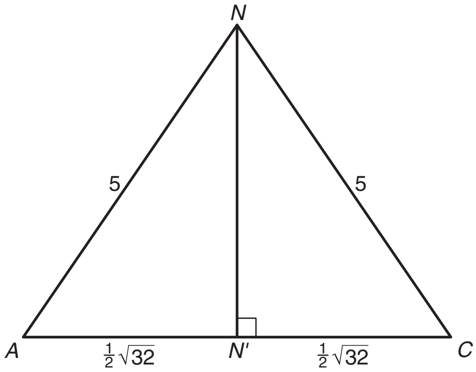
c De lengte van de rotatiebaan van M is gelijk aan de lengte van de baan van het punt P in de figuur bij a.
$PN = \sqrt{3^2 + 3^2} = \sqrt{18}$
Dus de lengte van de baan is $\frac{108{,}4}{360} \cdot 2\pi \cdot \sqrt{18} \approx 8{,}0$ dm.

10 a $d(M, EG) = MM'$ met M' op EG zo dat $MM' \perp EG$.



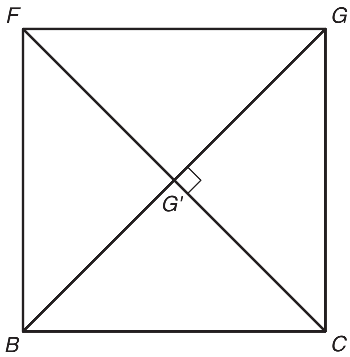
$FH = \sqrt{4^2 + 4^2} = \sqrt{32}$
$SH = \frac{1}{2} \cdot FH = \frac{1}{2}\sqrt{32}$
$MM' = \frac{1}{2} \cdot SH = \frac{1}{2} \cdot \frac{1}{2}\sqrt{32} = \frac{1}{4}\sqrt{32}$
$d(M, EG) = MM' = \frac{1}{4}\sqrt{32} \approx 1{,}41$

b



$d(N, AC) = NN'$ met N' op AC zo dat $NN' \perp AC$.
$NA = \sqrt{4^2 + 3^2} = 5$
$NC = \sqrt{4^2 + 3^2} = 5$
$AC = \sqrt{4^2 + 4^2} = \sqrt{32}$
$NN' = \sqrt{5^2 - (\frac{1}{2}\sqrt{32})^2} = \sqrt{25 - \frac{1}{4} \cdot 32} = \sqrt{17}$
$d(N, AC) = NN' = \sqrt{17} \approx 4{,}12$

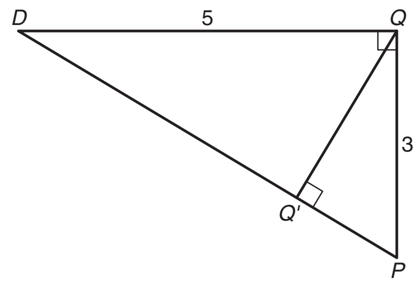
c $d(M, EDC) = d(GH, EDCF) = d(G, EDCF) = d(G, CF)$



$d(G, CF) = GG' = \frac{1}{2} \cdot GB = \frac{1}{2}\sqrt{32}$
Dus $d(M, EDC) = \frac{1}{2}\sqrt{32} \approx 2{,}83$.

11 a $d(PG, ABC) = d(G, ABC) = GC = 4$

b $d(Q, DP)$ in $\triangle DPQ$.



$PQ = 3$

$DQ = \sqrt{4^2 + 3^2} = 5$

$DP = \sqrt{5^2 + 3^2} = \sqrt{34}$

De zijde × hoogte-methode geeft

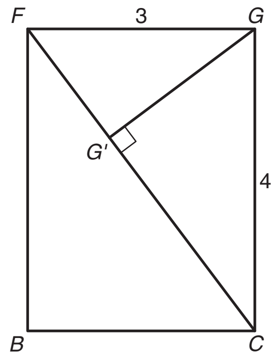
$DP \times QQ' = DQ \times PQ$

$\sqrt{34} \times QQ' = 5 \times 3$

$QQ' = \dfrac{15}{\sqrt{34}}$

Dus $d(Q, DP) = \dfrac{15}{\sqrt{34}} \approx 2{,}57$.

c $d(GH, EDC) = d(GH, EDCF) = d(G, EDCF) = d(G, CF)$



$FC = \sqrt{3^2 + 4^2} = 5$

De zijde × hoogte-methode in $\triangle CGF$ geeft

$FC \times GG' = CG \times FG$

$5 \times GG' = 4 \times 3$

$GG' = \frac{12}{5}$

Dus $d(GH, EDC) = \frac{12}{5}$.

d $d(BC, PQ) = d(P, BC) = BP = \sqrt{4^2 + 2^2} = \sqrt{20} \approx 4{,}47$