Diagnostische toets

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1 a $y = 3^x \xrightarrow{\text{translatie } (-2,0)} y = 3^{x+2}$

Er geldt $y = 3^{x+2} = 3^x \cdot 3^2 = 9 \cdot 3^x$.
Dus de vermenigvuldiging ten opzichte van de x-as met 9 levert dezelfde beeldfiguur op.

b $y = 3^x \xrightarrow{\text{verm. } x\text{-as met } \frac{1}{3}} y = \frac{1}{3} \cdot 3^x$

Er geldt $y = \frac{1}{3} \cdot 3^x = \dfrac{3^x}{3^1} = 3^{x-1}$

Dus de translatie (1, 0) levert dezelfde beeldfiguur op.

2 a ${}^2\log(9) + {}^2\log(11) = {}^2\log(9 \cdot 11) = {}^2\log(99)$

b $3 - {}^5\log(20) = {}^5\log(3^5) - {}^5\log(20) = {}^5\log\left(\dfrac{3^5}{20}\right) = {}^5\log(12{,}15)$

c $-2 + \log(18) = \log(10^{-2}) + \log(18) = \log(10^{-2} \cdot 18) = \log(0{,}18)$

d ${}^{\frac{1}{2}}\log(8) + {}^3\log(54) = {}^{\frac{1}{2}}\log((\tfrac{1}{2})^{-3}) + {}^3\log(54) = -3 + {}^3\log(54)$
$= {}^3\log(3^{-3}) + {}^3\log(54) = {}^3\log(3^{-3} \cdot 54) = {}^3\log(2)$

3 a $y = {}^3\log(x) \xrightarrow{\text{translatie } (0,-2)} y = -2 + {}^3\log(x)$

Er geldt $y = -2 + {}^3\log(x) = {}^3\log(3^{-2}) + {}^3\log(x) = {}^3\log(3^{-2} \cdot x) = {}^3\log(\frac{1}{9}x)$
Dus de vermenigvuldiging ten opzichte van de y-as met 9 levert dezelfde beeldfiguur op.

b $y = {}^3\log(x) \xrightarrow{\text{verm. } y\text{-as met } \frac{1}{27}} y = {}^3\log(27x)$

Er geldt $y = {}^3\log(27x) = {}^3\log(27) + {}^3\log(x) = {}^3\log(3^3) + {}^3\log(x) = 3 + {}^3\log(x)$.

Dus de translatie (0, 3) levert dezelfde beeldfiguur op.

4 $y = {}^2\log(4x) \xrightarrow{\text{translatie } (-2,3)} y = 3 + {}^2\log(4(x+2))$
Er geldt $g(x) = 3 + {}^2\log(4(x+2)) = 3 + {}^2\log(4) + {}^2\log(x+2)$
$= 3 + {}^2\log(2^2) + {}^2\log(x+2) = 3 + 2 + {}^2\log(x+2) = 5 + {}^2\log(x+2)$.
Dus $a = 5$ en $b = 2$.

5 a $3\,e^3 - 8\,e^3 = -5\,e^3$

b $5\,e^{5a} - 3\,e^{5a} = 2\,e^{5a}$

c $\dfrac{6\,e^{8x} + 3\,e^{4x}}{e^{2x}} = \dfrac{6\,e^{8x}}{e^{2x}} + \dfrac{3\,e^{4x}}{e^{2x}} = 6\,e^{6x} + 3\,e^{2x}$

d $(2\,e^a - 1)^2 = (2\,e^a - 1)(2\,e^a - 1) = (2\,e^a)^2 - 2\,e^a - 2\,e^a + 1 = 4\,e^{2a} - 4\,e^a + 1$

6 a $(x^2 - 4x - 5)\,e^x = 0$
$x^2 - 4x - 5 = 0 \vee e^x = 0$
$(x-5)(x+1) = 0$ geen opl.
$x - 5 = 0 \vee x + 1 = 0$
$x = 5 \vee x = -1$

b $e^{2x+1} - 1 = 0$
$e^{2x+1} = 1$
$e^{2x+1} = e^0$
$2x + 1 = 0$
$2x = -1$
$x = -\frac{1}{2}$

c $e^{x+1} - e^2 \cdot \sqrt{e} = 0$
$e^{x+1} = e^2 \cdot e^{\frac{1}{2}}$
$e^{x+1} = e^{2\frac{1}{2}}$
$x + 1 = 2\frac{1}{2}$
$x = 1\frac{1}{2}$

d $x^2\,e^{2x} = x\,e^{2x}$
$x^2\,e^{2x} - x\,e^{2x} = 0$
$(x^2 - x)\,e^{2x} = 0$
$x^2 - x = 0 \vee e^{2x} = 0$
$x(x-1) = 0$ geen opl.
$x = 0 \vee x - 1 = 0$
$x = 0 \vee x = 1$

7 a $f(x) = e^x + 2x^2 - 5x$ geeft $f'(x) = e^x + 4x - 5$

b Stel $y = e^{2x^2-5x} = e^u$ met $u = 2x^2 - 5x$

$$g'(x) = \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = e^u \cdot (4x-5) = e^{2x^2-5x} \cdot (4x-5)$$

c $h(x) = (2x^2 - 5x)\,e^x$ geeft
$h'(x) = [2x^2 - 5x]' \cdot e^x + (2x^2 - 5x) \cdot [\,e^x]'$
$= (4x - 5)\,e^x + (2x^2 - 5x)\,e^x$
$= (2x^2 - x - 5)\,e^x$

d $j(x) = 2x^2\,e^x - 5x$ geeft
$j'(x) = [2x^2]' \cdot e^x + 2x^2 \cdot [e^x]' - [5x]'$
$= 4x\,e^x + 2x^2\,e^x - 5$
$= (4x + 2x^2)\,e^x - 5$

8 a $f(x) = 0$ geeft $(2x - 3)\,e^{-x} = 0$
$2x - 3 = 0 \vee e^{-x} = 0$
$2x = 3$ geen opl.
$x = 1\frac{1}{2}$

Snijpunt van de grafiek met de x-as is het punt $(1\frac{1}{2}, 0)$.

$f'(x) = [2x-3]' \cdot e^{-x} + (2x-3) \cdot [e^{-x}]'$
$= 2\,e^{-x} + (2x-3) \cdot \boxed{e^{-x} \cdot -1}$
$= 2\,e^{-x} - (2x-3)\,e^{-x}$
$= (2 - 2x + 3)\,e^{-x}$
$= (5 - 2x)\,e^{-x}$

> De afgeleide van $y = e^{-x}$ met de kettingregel:
> Stel: $y = e^{-x} = e^u$ met $u = -x$
> $[e^{-x}]' = \frac{dy}{du} \cdot \frac{du}{dx} = e^u \cdot -1 = e^{-x} \cdot -1$

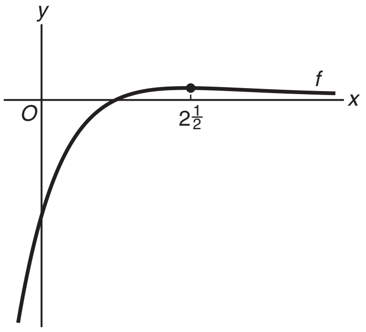
Stel raaklijn $y = ax + b$.

$$a = f'(1\tfrac{1}{2}) = (5 - 2 \cdot 1\tfrac{1}{2}) \cdot e^{-1\frac{1}{2}} = 2\,e^{-1\frac{1}{2}} = \frac{2}{e^{1\frac{1}{2}}} = \frac{2}{e\sqrt{e}}$$

dus $y = \frac{2}{e\sqrt{e}}x + b$
snijpunt $(1\frac{1}{2}, 0)$
$\Big\}$ $0 = \frac{2}{e\sqrt{e}} \cdot 1\frac{1}{2} + b$
$-\frac{3}{e\sqrt{e}} = b$

Dus raaklijn $y = \frac{2}{e\sqrt{e}}x - \frac{3}{e\sqrt{e}}$.

b $f'(x) = 0$ geeft $(5 - 2x)\,e^{-x} = 0$
$5 - 2x = 0 \vee e^{-x} = 0$
$-2x = -5$ geen opl.
$x = 2\frac{1}{2}$



max. is $f(2\frac{1}{2}) = (2 \cdot 2\frac{1}{2} - 3)\ e^{-2\frac{1}{2}} = \frac{2}{e^{2\frac{1}{2}}} = \frac{2}{e^2\sqrt{e}}$

Dus top$\left(2\frac{1}{2}, \frac{2}{e^2\sqrt{e}}\right)$.

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9 a $e^x = 4$
$x = \ln(4)$
b $e^{x-2} = 3$
$x - 2 = \ln(3)$
$x = 2 + \ln(3)$
c $e^{\frac{1}{2}x} = 5$
$\frac{1}{2}x = \ln(5)$
$x = 2\ln(5)$
d $\ln(x) = -2$
$x = e^{-2} = \frac{1}{e^2}$
e $\ln(2 - x) = 4$
$2 - x = e^4$
$-x = -2 + e^4$
$x = 2 - e^4$
f $\ln^2(x) = 25$
$\ln(x) = -5 \vee \ln(x) = 5$
$x = e^{-5} = \frac{1}{e^5} \vee x = e^5$
g $\ln(x^2) = 6$
$x^2 = e^6$
$x = -\sqrt{e^6} \vee x = \sqrt{e^6}$
$x = -(e^6)^{\frac{1}{2}} \vee x = (e^6)^{\frac{1}{2}}$
$x = -e^3 \vee x = e^3$
$x = \frac{1}{e^3} \vee x = e^3$
h $e^{2x} \cdot \ln(\frac{1}{2}x) = 0$
$e^{2x} = 0 \vee \ln(\frac{1}{2}x) = 0$
$\frac{1}{2}x = e^0 = 1$
geen opl. $x = 2$
i $(x - e)\ln(x) = 0$
$x - e = 0 \vee \ln(x) = 0$
$x = e \vee x = e^0 = 1$

10 a Stel $y = 3^{4x-1} = 3^u$ met $u = 4x - 1$
$f'(x) = \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = 3^u \cdot \ln(3) \cdot 4 = 3^{4x-1} \cdot \ln(3) \cdot 4$
$= 4 \cdot 3^{4x-1} \cdot \ln(3)$
b Stel $y = 5 \cdot 2^{3x} = 5 \cdot 2^u$ met $u = 3x$
$g'(x) = \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = 5 \cdot 2^u \cdot \ln(2) \cdot 3 = 15 \cdot 2^{3x} \cdot \ln(2)$
c $h(x) = x^2 \cdot 4^x$ geeft $h'(x) = [x^2]' \cdot 4^x + x^2 \cdot [4^x]'$
$= 2x \cdot 4^x + x^2 \cdot 4^x \cdot \ln(4)$

11 a $f(x) = \ln(e\ x) = \ln(e) + \ln(x) = 1 + \ln(x)$ geeft $f'(x) = \frac{1}{x}$
b $g(x) = e \cdot \ln(3x)$ geeft
$g'(x) = e \cdot [\ln(3x)]' = e \cdot [\ln(3) + \ln(x)]' = e \cdot \frac{1}{x} = \frac{e}{x}$

c $h(x) = x^2 + \ln(x^2) = x^2 + 2\ln(x)$ geeft

$h'(x) = 2x + 2 \cdot \frac{1}{x} = 2x + \frac{2}{x}$

d $j(x) = x^2 \cdot {}^2\log(x)$ geeft

$j'(x) = [x^2]' \cdot {}^2\log(x) + x^2 \cdot [{}^2\log(x)]'$

$= 2x \cdot {}^2\log + x^2 \cdot \frac{1}{x\ln(2)} = 2x \cdot {}^2\log(x) + \frac{x}{\ln(2)}$

e $k(x) = \ln\left(\frac{6}{x}\right) = \ln(6) - \ln(x)$ geeft $k'(x) = -\frac{1}{x}$

f $l(x) = {}^4\log(4x) = {}^4\log(4) + {}^4\log(x)$ geeft $l'(x) = \frac{1}{x\ln(4)}$

12 a Domein: $\langle 0, \rightarrow \rangle$

b $f(x) = 0$ geeft $x^2 \cdot \ln(x) = 0$

$x^2 = 0 \vee \ln(x) = 0$

$\underbrace{x = 0}_{\text{voldoet niet}} \vee x = e^0 = 1$

Snijpunt met de x-as is het punt $(1, 0)$.

$f(x) = x^2 \cdot \ln(x)$ geeft

$f'(x) = [x^2]' \cdot \ln(x) + x^2 \cdot [\ln(x)]' = 2x \cdot \ln(x) + x^2 \cdot \frac{1}{x}$

$= 2x \cdot \ln(x) + x$

Stel raaklijn k: $y = ax + b$

$a = f'(1) = 2 \cdot 1 \cdot \ln(1) + 1 = 1$

dus k: $y = x + b$ }
snijpunt $(1, 0)$ } $0 = 1 + b$

$-1 = b$

Dus k: $y = x - 1$.

c $f'(x) = 0$ geeft $2x \cdot \ln(x) + x = 0$

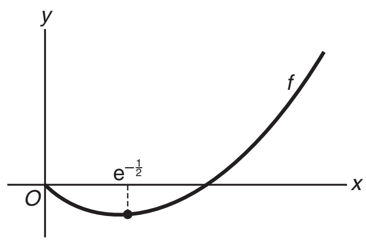
$x(2\ln(x) + 1) = 0$

$\underbrace{x = 0}_{\text{voldoet niet}} \vee 2\ln(x) + 1 = 0$

$2\ln(x) = -1$

$\ln(x) = -\frac{1}{2}$

$x = e^{-\frac{1}{2}} = \frac{1}{\sqrt{e}}$



Min. is $f\left(\frac{1}{\sqrt{e}}\right) = f(e^{-\frac{1}{2}}) = (e^{-\frac{1}{2}})^2 \cdot \ln(e^{-\frac{1}{2}}) = e^{-1} \cdot -\frac{1}{2} = -\frac{1}{2e}$

13 a $y = e^x \xrightarrow{\text{verm. t.o.v. } x\text{-as met } 2} y = 2\,e^x$

Er geldt $y = 2\,e^x = e^{\ln(2)} \cdot e^x = e^{x+\ln(2)}$.

Dus de translatie $(-\ln(2), 0)$ levert dezelfde beeldfiguur op.

b $y = \ln(x) \xrightarrow{\text{verm. t.o.v. } y\text{-as met } 2} y = \ln(\frac{1}{2}x)$

Er geldt $y = \ln(\frac{1}{2}x) = \ln(\frac{1}{2}) + \ln(x)$.

Dus de translatie $(0, \ln(\frac{1}{2}))$ levert dezelfde beeldfiguur op.

14 $y = 3\ln(x) \xrightarrow{\text{translatie } (0,2)} y = 3\ln(x) + 2 \xrightarrow{\text{verm. t.o.v } y\text{-as met } 2} y = 3\ln(\tfrac{1}{2}x) + 2$

Er geldt $y = 3\ln(\tfrac{1}{2}x) + 2 = \ln((\tfrac{1}{2}x)^3) + \ln(\mathrm{e}^2)$

$= \ln((\tfrac{1}{2}x)^3 \cdot \mathrm{e}^2) = \ln\left(\frac{\mathrm{e}^2}{8} \cdot x^3\right)$

Dus $a = \frac{\mathrm{e}^2}{8}$ en $b = 3$.

15 $p = 18\ln(q) - 15$

$18\ln(q) = p + 15$

$\ln(q) = \frac{p}{18} + \frac{15}{18}$

$q = \mathrm{e}^{\frac{p}{18} + \frac{5}{6}}$

$q = \mathrm{e}^{\frac{p}{18}} \cdot \mathrm{e}^{\frac{5}{6}}$

$q = \mathrm{e}^{\frac{5}{6}} \cdot (\mathrm{e}^{\frac{1}{18}})^p$

$q \approx 2{,}301 \cdot 1{,}057^p$

Dus $a \approx 2{,}301$ en $g \approx 1{,}057$.

16 a $F = 16(0{,}6 + \ln(100)) \approx 83$ slagen per minuut.

b $78 = 16(0{,}6 + \ln(G))$

$\frac{78}{16} = 0{,}6 + \ln(G)$

$\ln(G) = 4{,}275$

$G = \mathrm{e}^{4{,}275} \approx 72$ kg

c $F = 16(0{,}6 + \ln(G))$

$\frac{F}{16} = 0{,}6 + \ln(G)$

$\ln(G) = \frac{F}{16} - 0{,}6$

$G = \mathrm{e}^{\frac{F}{16} - 0{,}6}$

$G = \frac{\mathrm{e}^{\frac{F}{16}}}{\mathrm{e}^{0{,}6}}$

$G = \frac{1}{\mathrm{e}^{0{,}6}} \cdot (\mathrm{e}^{\frac{1}{16}})^F$

$G \approx 0{,}549 \cdot 1{,}064^F$

Dus $b \approx 0{,}549$ en $g \approx 1{,}064$.